Problem Sheet 2
Solid State Theory
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Problem 1) **(4 Points)**

In a quantum solid the dominant repulsive energy is the zero-point energy of the atoms. Consider a crude one-dimensional model of ^{4}He with each He atom confined to a line segement of length L. In the ground state the wave function within each segment is taken as a half wavelength of a free particle. Find the zero-point kinetic energy per particle.

Problem 2) **(4 Points)**

Consider a longitudinal wave

$$u(na,t) = u \cdot \cos(\omega t - kna),$$

which propagates in a monatomic linear lattice of atoms with mass M, lattice constant a and nearest neighbour interaction C.

(a) Show that the total energy of the wave is

$$E = \frac{M}{2}\sum_n \left[\frac{\mathrm{d}}{\mathrm{d}t}u(na,t)\right]^2 + \frac{C}{2}\sum_n [u(na,t) - u((n+1)a,t)]^2.$$

(b) By substituting $u(na,t)$ in the expression, show that the time averaged total energy per atom is $(M/2)\omega^2 u^2$ with the help of the dispersion relation (22.29) from the lecture.

Problem 3) **(4 Points)**

Consider the normal modes of a linear chain in which the force constants between nearest-neighbour atoms are alternately C and $10 \cdot C$. Let the masses be equal, and let the nearest-neighbour separation be $a/2$. Find $\omega(k)$ at $k = 0$ and at $k = \pi/a$. Sketch the dispersion relation by hand.