



Capillary pressure, hysteresis and residual saturation in porous media

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Abstract

A macroscopic theory for capillarity in porous media is presented. The capillary pressure function in this theory is not an input parameter but an outcome. The theory is based on introducing the trapped or residual saturations as state variables. It allows to predict spatiotemporal changes in residual saturation. The theory yields process dependence and hysteresis in capillary pressure as its main result.

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1. Introduction and formulation of the problem

A predictive macroscopic theory of two-phase fluid flow inside a rigid porous medium is a longstanding problem in the physics of fluids, soft matter, and disordered systems [1–3]. Describing or predicting the flow of two immiscible and incompressible fluids is of fundamental practical importance for numerous applied

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sciences such as geophysics, hydrology, hydrocarbon production, filtration technology, and many other applied fields [4–7].

My objective in this short paper is to introduce a constitutive theory for macroscopic capillarity based on the insight that trapped and residual fluids are important [8–10]. A central motivation for this work are the well known difficulties surrounding the traditional macroscopic equations that are based on the concepts of capillary pressure and relative permeabilities (see Refs. [4,11]). In particular nonuniqueness of the capillary pressure hampers its use when drainage and imbibition occur simultaneously (as, e.g. during infiltration of a drop of liquid into a porous medium [12]). Other well known problems with the capillary pressure are its hysteresis, process dependence and dynamic effects such as dependence on velocities or rates of saturation change. Residual saturations are not constant as assumed in the traditional macroscopic theory. Experimental observations show instead that residual saturations vary as functions of position and time. Most scientists and engineers ignore these problems and continue to use the traditional set of equations.

Distinguishing between percolating and nonpercolating (trapped) fluid regions is important for two-phase immiscible displacement. Experimentally the difference between percolating and trapped fluids becomes apparent from the fact that hydrostatic equilibrium pressures propagate only in the percolating fluid regions. In [8–10] a theoretical formulation was introduced that incorporates this distinction.

Given the basic distinction between percolating and nonpercolating fluids the present paper develops a comprehensive constitutive theory based on the ideas in [8–10]. Let me summarize the content of this paper. Once the definition of percolating versus nonpercolating regions has been given in Section 2 the general balance laws for mass, momentum and volume are formulated in Section 3. Refs. [8–10] have also discussed energy balance, but this will be unnecessary in the present approach. In Section 4 the complete list of constitutive assumptions is given. An analysis of the resulting equations of motion in special limiting cases of experimental relevance shows that the traditional concept of capillary pressure is a consequence (not an input) of the present theory. My presentation concludes by identifying a closed expression for capillary pressure and discussing its limits of validity in Section 5.

2. Percolating versus nonpercolating fluid regions

This section defines the difference between percolating and nonpercolating fluid regions on the pore scale. The physical importance of this difference arises from the fact that in static equilibrium the pressure can become hydrostatic only in those fluid regions that are connected (or percolating) to the sample boundary.

Consider a sample $\mathbb{S} = \mathbb{P} \cup \mathbb{M}$ consisting of a solid (and rigid) matrix \mathbb{M} and fluid filled pores \mathbb{P} . The boundary of the sample region $\mathbb{S} \subset \mathbb{R}^3$ is denoted as $\partial\mathbb{S}$. The pore space \mathbb{P} is occupied by two fluids, a wetting fluid called water \mathbb{W} and a nonwetting fluid called oil \mathbb{O} (or air). Each fluid consists of disjoint and pathconnected subsets

(regions) $\mathbb{W}_i, \mathbb{O}_i$. More precisely one has

$$\mathbb{W} = \bigcup_{i=1}^{N_{\mathbb{W}}} \mathbb{W}_i, \tag{1a}$$

$$\mathbb{O} = \bigcup_{i=1}^{N_{\mathbb{O}}} \mathbb{O}_i, \tag{1b}$$

where the subsets $\mathbb{W}_i, \mathbb{O}_i$ are mutually disjoint but each of them is pathconnected. A set is called pathconnected if any two of its points can be connected by a path contained inside the set. The sets are called mutually disjoint if $\mathbb{O}_i \cap \mathbb{O}_j = \emptyset$ and $\mathbb{W}_i \cap \mathbb{W}_j = \emptyset$ holds for all $i \neq j$. The numbers $N_{\mathbb{W}}, N_{\mathbb{O}}$ give the total number of pathconnected subsets for water and oil. Of course, $N_{\mathbb{W}}, N_{\mathbb{O}}$ as well as the regions $\mathbb{W}_i, \mathbb{O}_i$ change with time during the displacement process.

Now define percolating ($\mathbb{F}_1, \mathbb{F}_3$) and nonpercolating ($\mathbb{F}_2, \mathbb{F}_4$) subsets by classifying the subsets as to whether they have empty or nonempty intersection with the sample boundary $\partial\mathbb{S}$. More formally define

$$\mathbb{F}_1 = \bigcup_{\substack{i=1 \\ \partial\mathbb{W}_i \cap \partial\mathbb{S} \neq \emptyset}}^{N_{\mathbb{W}}} \mathbb{W}_i, \tag{2a}$$

$$\mathbb{F}_2 = \bigcup_{\substack{i=1 \\ \partial\mathbb{W}_i \cap \partial\mathbb{S} = \emptyset}}^{N_{\mathbb{W}}} \mathbb{W}_i, \tag{2b}$$

$$\mathbb{F}_3 = \bigcup_{\substack{i=1 \\ \partial\mathbb{O}_i \cap \partial\mathbb{S} \neq \emptyset}}^{N_{\mathbb{O}}} \mathbb{O}_i, \tag{2c}$$

$$\mathbb{F}_4 = \bigcup_{\substack{i=1 \\ \partial\mathbb{O}_i \cap \partial\mathbb{S} = \emptyset}}^{N_{\mathbb{O}}} \mathbb{O}_i, \tag{2d}$$

where for (\mathbb{F}_1) resp. (\mathbb{F}_3) the region \mathbb{W}_i resp. (\mathbb{O}_i) have nonempty intersection with the sample boundary $\partial\mathbb{S}$ while for (\mathbb{F}_2) resp. \mathbb{F}_4 they do not. In this way each point in \mathbb{P} belongs to one of four regions $\mathbb{F}_i, i = 1, 2, 3, 4$. This results in a total of four fluid phases called percolating (resp. nonpercolating) water, and percolating (resp. nonpercolating) oil. The index $i = 5$ will be used for the rigid matrix (= rock).

3. Balance laws

From now on the discussion refers to a macroscopic scale much larger than the pore scale. The microscopic details of the pore scale geometry inside the sample region \mathbb{S} are considered to be smeared out, and the configuration of pores and fluids is described by macroscopic quantities.

The first of these macroscopic quantities is the average porosity $\phi(\mathbf{x})$ defined as the (possibly position dependent) volume fraction of pore space \mathbb{P} in a region centered

at \mathbf{x} . The volume fractions of the subsets $\mathbb{F}_i \subset \mathbb{S}$, $i = 1, 2, 3, 4$ and $\mathbb{M} \subset \mathbb{S}$ are denoted as $\phi_i(\mathbf{x}, t)$. They are in general position and time dependent. Volume conservation requires the relations

$$\phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 = 1, \quad (3a)$$

$$S_1 + S_2 + S_3 + S_4 = 1, \quad (3b)$$

$$1 - \phi = \phi_5, \quad (3c)$$

where $\phi_i(\mathbf{x}, t) = \phi(\mathbf{x})S_i(\mathbf{x}, t)$ ($i = 1, 2, 3, 4$) are volume fractions, and $S_i(\mathbf{x}, t)$ are the saturations of the four phases. The important difference between percolating and nonpercolating fluid regions on the pore scale is reflected macroscopically by introducing four instead of two saturation fields $S_i(\mathbf{x}, t)$. Finally

$$S_{\mathbb{W}} = S_1 + S_2, \quad (4a)$$

$$S_{\mathbb{D}} = S_3 + S_4, \quad (4b)$$

defines the total wetting and nonwetting fluid saturations $S_{\mathbb{W}}, S_{\mathbb{D}}$, respectively,

Mass balance for the four phases requires

$$\frac{\partial(\phi_1 \varrho_1)}{\partial t} + \nabla \cdot (\phi_1 \varrho_1 \mathbf{v}_1) = M_1, \quad (5a)$$

$$\frac{\partial(\phi_2 \varrho_2)}{\partial t} + \nabla \cdot (\phi_2 \varrho_2 \mathbf{v}_2) = M_2, \quad (5b)$$

$$\frac{\partial(\phi_3 \varrho_3)}{\partial t} + \nabla \cdot (\phi_3 \varrho_3 \mathbf{v}_3) = M_3, \quad (5c)$$

$$\frac{\partial(\phi_4 \varrho_4)}{\partial t} + \nabla \cdot (\phi_4 \varrho_4 \mathbf{v}_4) = M_4, \quad (5d)$$

where \mathbf{v}_i ($i = 1, 2, 3, 4$) are the velocities of the four phases, ϱ_i are the densities and M_i the mass transfer rates into phase i from all the other phases.

Momentum balance is generally formulated as

$$\phi_1 \varrho_1 \left(\frac{\partial}{\partial t} + \mathbf{v}_1 \cdot \nabla \right) \mathbf{v}_1 - \phi_1 \nabla \cdot \Sigma_1 - \phi_1 \mathbf{F}_1 = \mathbf{m}_1 - \mathbf{v}_1 M_1, \quad (6a)$$

$$\phi_2 \varrho_2 \left(\frac{\partial}{\partial t} + \mathbf{v}_2 \cdot \nabla \right) \mathbf{v}_2 - \phi_2 \nabla \cdot \Sigma_2 - \phi_2 \mathbf{F}_2 = \mathbf{m}_2 - \mathbf{v}_2 M_2, \quad (6b)$$

$$\phi_3 \varrho_3 \left(\frac{\partial}{\partial t} + \mathbf{v}_3 \cdot \nabla \right) \mathbf{v}_3 - \phi_3 \nabla \cdot \Sigma_3 - \phi_3 \mathbf{F}_3 = \mathbf{m}_3 - \mathbf{v}_3 M_3, \quad (6c)$$

$$\phi_4 \varrho_4 \left(\frac{\partial}{\partial t} + \mathbf{v}_4 \cdot \nabla \right) \mathbf{v}_4 - \phi_4 \nabla \cdot \Sigma_4 - \phi_4 \mathbf{F}_4 = \mathbf{m}_4 - \mathbf{v}_4 M_4, \quad (6d)$$

where Σ_i is the stress tensor in the i th phase, \mathbf{F}_i is the body force per unit volume acting on the i th phase and \mathbf{m}_i is the momentum transfer into phase i from all the other phases.

4. Constitutive assumptions

The general balance laws need to be augmented with constitutive assumptions. They are listed in this section.

If the porous medium is macroscopically homogeneous its porosity

$$\phi(\mathbf{x}) = \phi = \text{const.} \quad (7)$$

If the fluids are incompressible then their densities are given as

$$\varrho_1(\mathbf{x}, t) = \varrho_{\mathbb{W}}, \quad (8a)$$

$$\varrho_2(\mathbf{x}, t) = \varrho_{\mathbb{W}}, \quad (8b)$$

$$\varrho_3(\mathbf{x}, t) = \varrho_{\mathbb{O}}, \quad (8c)$$

$$\varrho_4(\mathbf{x}, t) = \varrho_{\mathbb{O}}, \quad (8d)$$

and they are constants $\varrho_{\mathbb{W}}, \varrho_{\mathbb{O}}$ independent of \mathbf{x} and t . When chemical reactions between the fluids are absent or neglected, one has

$$M_2 = -M_1, \quad (9a)$$

$$M_4 = -M_3, \quad (9b)$$

to ensure mass conservation for water and oil. For sufficiently slow flows the nonlinear inertial terms

$$(\mathbf{v}_i \cdot \nabla) \mathbf{v}_i = 0, \quad (10)$$

and the accelerations

$$\frac{\partial}{\partial t} \mathbf{v}_i = 0, \quad (11)$$

may be neglected. The body forces are gravity and capillary forces

$$\mathbf{F}_1 = \varrho_1 \mathbf{g}, \quad (12a)$$

$$\mathbf{F}_2 = \varrho_2 \mathbf{g} + \mathbf{F}_{c\mathbb{W}}, \quad (12b)$$

$$\mathbf{F}_3 = \varrho_3 \mathbf{g}, \quad (12c)$$

$$\mathbf{F}_4 = \varrho_4 \mathbf{g} + \mathbf{F}_{c\mathbb{O}}, \quad (12d)$$

with \mathbf{g} being the acceleration due to gravity. The capillary body forces $\mathbf{F}_{c\mathbb{W}}, \mathbf{F}_{c\mathbb{O}}$ are introduced to describe the effects of capillarity and these forces are experimentally evident through the presence of nonpercolating phases. Depending on the wetting properties and pore structure of the medium, the capillary body forces keep trapped fluids in place.

The momentum transfer into phase i from all the other phases is assumed to be a simple viscous drag as it is assumed also in the traditional theory. Thus

$$\mathbf{m}_i = \sum_{j=1}^5 R_{ij}(\mathbf{v}_j - \mathbf{v}_i), \quad (13)$$

where the resistance coefficient R_{ij} quantifies the viscous coupling between phase i and j . It follows from reciprocity that

$$R_{ij} = R_{ji}. \quad (14)$$

For the rigid rock matrix $\mathbf{v}_5 = 0$ and hence $-R_{i5}\mathbf{v}_i$ is the momentum transfer from the wall into phase i . Then

$$\mathbf{m}_1 = R_{13}(\mathbf{v}_3 - \mathbf{v}_1) + R_{14}(\mathbf{v}_4 - \mathbf{v}_1) - R_{15}\mathbf{v}_1, \quad (15a)$$

$$\mathbf{m}_2 = R_{23}(\mathbf{v}_3 - \mathbf{v}_2) + R_{24}(\mathbf{v}_4 - \mathbf{v}_2) - R_{25}\mathbf{v}_2, \quad (15b)$$

$$\mathbf{m}_3 = R_{31}(\mathbf{v}_1 - \mathbf{v}_3) + R_{32}(\mathbf{v}_2 - \mathbf{v}_3) - R_{35}\mathbf{v}_3, \quad (15c)$$

$$\mathbf{m}_4 = R_{41}(\mathbf{v}_1 - \mathbf{v}_4) + R_{42}(\mathbf{v}_2 - \mathbf{v}_4) - R_{45}\mathbf{v}_4, \quad (15d)$$

where $R_{12} = 0$ and $R_{34} = 0$ were used because there is no common interface and hence no direct viscous interaction between these phase pairs. Each R_{ij} is a 3×3 -matrix.

The capillary body forces are specified as gradients of capillary potentials

$$\mathbf{F}_{c\mathbb{W}} = -\nabla\Pi_{c\mathbb{W}}, \quad (16a)$$

$$\mathbf{F}_{c\mathbb{O}} = -\nabla\Pi_{c\mathbb{O}}, \quad (16b)$$

where the capillary potentials $\Pi_{c\mathbb{W}}, \Pi_{c\mathbb{O}}$ are defined as

$$\Pi_{c\mathbb{W}} = \Pi_a^* - \Pi_a S_1^{-\alpha}, \quad (17a)$$

$$\Pi_{c\mathbb{O}} = \Pi_b^* - \Pi_b S_3^{-\beta}, \quad (17b)$$

with constants $\Pi_a^*, \Pi_b^*, \Pi_a, \Pi_b$ and exponents $\alpha, \beta > 0$. The capillary body force is large where the saturation gradients are large and the percolating saturations are small.

As in the traditional theory the off-diagonal components of the stress tensors Σ_i of the two fluids are neglected. For the percolating phases one has

$$\Sigma_1 = -P_1\mathbf{1}, \quad (18)$$

$$\Sigma_3 = -P_3\mathbf{1}, \quad (19)$$

where P_1 and P_3 are the fluid pressures. For the nonpercolating phases the stress tensors Σ_2, Σ_4 cannot be specified in this way because the forces (resp. hydrostatic pressure) cannot propagate in these phase regions. Here it is assumed that these stresses are given by the pressure in the surrounding percolating phase modified by the energy density stored in the common interface with the surrounding percolating phases. This suggests an Ansatz [13]:

$$\Sigma_2 = -P_3\mathbf{1} + \frac{\sigma_{\mathbb{W}\mathbb{O}}}{\phi} \frac{\partial A_{32}}{\partial S_2} \mathbf{1}, \quad (20a)$$

$$\Sigma_4 = -P_1\mathbf{1} + \frac{\sigma_{\mathbb{W}\mathbb{O}}}{\phi} \frac{\partial A_{41}}{\partial S_4} \mathbf{1}, \quad (20b)$$

where $\sigma_{\mathbb{W}\mathbb{O}}$ is the oil–water interfacial tension, and the unknowns $A_{32}(\mathbf{x}, t)$, $A_{41}(\mathbf{x}, t)$ resp. are the interfacial areas per unit volume of porous medium between phases 3

and 2, (resp. 4 and 1). (The factor ϕ arises from the definition of A_{ij} per unit volume of porous medium.) To avoid the necessity of finding equations of motion for the new unknowns A_{32} and A_{41} it is assumed that geometrical relations of the form

$$A_{32} = A_2^* S_2^\gamma, \tag{21a}$$

$$A_{41} = A_4^* S_4^\delta, \tag{21b}$$

hold, where A_2^*, A_4^* are prefactors assumed to be constant. In summary, the expressions

$$\Sigma_2 = (-P_3 + \gamma P_2^* S_2^{\gamma-1}) \mathbf{1}, \tag{22a}$$

$$\Sigma_4 = (-P_1 + \delta P_4^* S_4^{\delta-1}) \mathbf{1}, \tag{22b}$$

for the stress tensors will be used below. Here

$$P_2^* = A_2^* \frac{\sigma_{\text{W}\text{O}}}{\phi}, \tag{23a}$$

$$P_4^* = A_4^* \frac{\sigma_{\text{W}\text{O}}}{\phi}, \tag{23b}$$

are constants. The mass transfer rates must depend on the rates of saturation change. For homogeneous media and incompressible fluids it is assumed that

$$M_{12} = \eta_2 \phi \rho_{\text{W}} \left(\frac{S_2 - S_2^*}{S_{\text{W}}^* - S_{\text{W}}} \right) \frac{\partial S_{\text{W}}}{\partial t}, \tag{24a}$$

$$M_{34} = \eta_4 \phi \rho_{\text{O}} \left(\frac{S_4 - S_4^*}{S_{\text{O}}^* - S_{\text{O}}} \right) \frac{\partial S_{\text{O}}}{\partial t}, \tag{24b}$$

where the real numbers η_2, η_4 are constant parameters. The limiting saturations are defined by

$$S_{\text{W}}^* = \frac{(1 - S_{\text{O im}})}{2} \left[1 + \tanh \left(\tau_{\text{W}} \frac{\partial S_{\text{W}}}{\partial t} \right) \right] + \frac{S_{\text{W dr}}}{2} \left[1 - \tanh \left(\tau_{\text{W}} \frac{\partial S_{\text{W}}}{\partial t} \right) \right], \tag{25a}$$

$$S_{\text{O}}^* = \frac{(1 - S_{\text{W dr}})}{2} \left[1 + \tanh \left(\tau_{\text{O}} \frac{\partial S_{\text{O}}}{\partial t} \right) \right] + \frac{S_{\text{O im}}}{2} \left[1 - \tanh \left(\tau_{\text{O}} \frac{\partial S_{\text{O}}}{\partial t} \right) \right], \tag{25b}$$

$$S_2^* = \frac{S_{\text{W dr}}}{2} \left[1 - \tanh \left(\tau_2 \frac{\partial S_{\text{W}}}{\partial t} \right) \right], \tag{25c}$$

$$S_4^* = \frac{S_{\text{O im}}}{2} \left[1 - \tanh \left(\tau_4 \frac{\partial S_{\text{O}}}{\partial t} \right) \right], \tag{25d}$$

where the parameters $S_{\text{W dr}}, S_{\text{O im}}$ characterize the wetting properties of the porous medium. The times $\tau_{\text{W}}, \tau_{\text{O}}, \tau_2, \tau_4$ are equilibration time scales for reaching capillary

equilibrium. For simplicity it will be assumed that there exists a single time scale

$$\tau = \tau_{\text{W}} = \tau_{\text{O}} = \tau_2 = \tau_4, \quad (26)$$

for capillary equilibration.

5. Results and discussion

The equations of motion are obtained by inserting the constitutive assumptions into the general balance laws. The resulting set of coupled equations can be analysed in certain special cases. Note that the formulation above does not contain a capillary pressure-saturation relationship $P_c(S_{\text{W}})$ as an input. This contrasts with the traditional theory (see Ref. [3]) where such a relation is needed as a given input parameter.

The central objective in this paper is to show that, under certain conditions, the formulation above yields a capillary pressure function $P_c(S_{\text{W}})$ as a result. The most important of these conditions is the assumption that the motion of the residual (nonpercolating) fluid phases $i = 2, 4$ decouples from the flow of the percolating fluid phases $i = 1, 3$. This is tacitly assumed in the traditional theory. The corresponding approximation may be called *residual decoupling approximation* and it can be formulated mathematically as $\mathbf{v}_4 = \mathbf{0}, \mathbf{v}_2 = \mathbf{0}$ and $R_{23} = 0, R_{41} = 0$. In the residual decoupling approximation the equations above reduce to a system of 17 equations for 12 unknowns ($P_1, P_3, \mathbf{v}_1, \mathbf{v}_3$ and $S_i, i = 1, 2, 3, 4$).

The main result of this paper gives an explicit formula for capillary pressure $P_c(S_{\text{W}})$ together with conditions for its validity. The capillary pressure is identified in the present formulation by $P_c = P_3 - P_1$ and subsequently evaluated by comparison with the formula $P_c(S_{\text{W}}) = C + (q_{\text{O}} - q_{\text{W}})\mathbf{g} \cdot \mathbf{x}$ that holds for capillary equilibrium in the traditional theory [3,4]. With this the equilibrium capillary pressure is obtained after some calculation as

$$P_c(S_{\text{W}}) = \frac{1}{2} \left(C + \frac{\Pi_{\text{a}}}{(S_{\text{W}} - S_2)^{\alpha}} - \frac{\Pi_{\text{b}}}{(1 - S_{\text{W}} - S_4)^{\beta}} + \gamma P_2^* S_2^{\gamma-1} - \delta P_4^* S_4^{\delta-1} \right), \quad (27)$$

where C is an integration constant that may in general depend on time t . The saturations S_2, S_4 for the nonpercolating fluids are given as functions of S_{W} by

$$S_2(\mathbf{x}, t) = S_2^*(\mathbf{x}) + (S_{20}(\mathbf{x}) - S_2^*(\mathbf{x})) \left(\frac{S_{\text{W}}^*(\mathbf{x}) - S_{\text{W}}(\mathbf{x}, t)}{S_{\text{W}}^*(\mathbf{x}) - S_{\text{W}0}(\mathbf{x})} \right)^{\eta_2}, \quad (28a)$$

$$S_4(\mathbf{x}, t) = S_4^*(\mathbf{x}) + (S_{40}(\mathbf{x}) - S_4^*(\mathbf{x})) \left(\frac{S_{\text{W}}(\mathbf{x}, t) - S_{\text{W}}^*(\mathbf{x})}{S_{\text{W}0}(\mathbf{x}) - S_{\text{W}}^*(\mathbf{x})} \right)^{\eta_4}, \quad (28b)$$

where

$$S_{\text{W}}(\mathbf{x}, t_0) = S_{\text{W}0}(\mathbf{x}), \quad (29a)$$

$$S_2(\mathbf{x}, t_0) = S_{20}(\mathbf{x}), \quad (29b)$$

$$S_4(\mathbf{x}, t_0) = S_{40}(\mathbf{x}), \quad (29c)$$

are the initial saturations for the displacement process. The limiting saturations $S_{\mathbb{W}}^*$, $S_{\mathbb{O}}^*$, S_2^* and S_4^* are given by

$$S_{\mathbb{W}}^*(\mathbf{x}) = 1 - S_{\mathbb{O} \text{ im}}(\mathbf{x}), \quad (30a)$$

$$S_{\mathbb{O}}^*(\mathbf{x}) = S_{\mathbb{O} \text{ im}}(\mathbf{x}), \quad (30b)$$

$$S_2^*(\mathbf{x}) = 0, \quad (30c)$$

$$S_4^*(\mathbf{x}) = S_{\mathbb{O} \text{ im}}(\mathbf{x}), \quad (30d)$$

for imbibition processes (i.e. $\partial S_{\mathbb{W}}/\partial t > 0$), and by

$$S_{\mathbb{W}}^*(\mathbf{x}) = S_{\mathbb{W} \text{ dr}}(\mathbf{x}), \quad (31a)$$

$$S_{\mathbb{O}}^*(\mathbf{x}) = 1 - S_{\mathbb{W} \text{ dr}}(\mathbf{x}), \quad (31b)$$

$$S_2^*(\mathbf{x}) = S_{\mathbb{W} \text{ dr}}(\mathbf{x}), \quad (31c)$$

$$S_4^*(\mathbf{x}) = 0, \quad (31d)$$

for drainage processes (i.e. $\partial S_{\mathbb{W}}/\partial t < 0$).

Eq. (27) for P_c does not hold generally, but only in the residual decoupling approximation together with additional simplifying approximations. The approximations under which Eq. (27) has been derived may be viewed as the conditions of validity for the traditional theory. The first approximation is the residual decoupling approximation introduced above. It reflects the fact that the traditional theory does not allow changes of the residual saturations.

To proceed further a second approximation is needed. It assumes that the time scale for equilibration of the interface configuration is very large, i.e. that $\tau \gg \partial S_{\mathbb{W}}/\partial t$ holds. This simplifies the mass transfer terms.

Thirdly, one also assumes viscous decoupling, i.e. that $R_{31} = 0$ and $R_{13} = 0$ holds. This assumption is also made in the traditional theory [3]. In addition one needs $R_{13} + R_{14} + R_{15} \gg M_{12}$ and $R_{31} + R_{32} + R_{35} \gg M_{34}$. Finally it needs to be assumed that the velocities and volume fractions are independent of \mathbf{x} , i.e. that $\mathbf{v}_1 = \text{const}$, $\mathbf{v}_3 = \text{const}$, $\phi_1 = \text{const}$, $\phi_3 = \text{const}$, and that the velocities are very small, i.e. that $\mathbf{v}_1 \rightarrow 0$, $\mathbf{v}_3 \rightarrow 0$.

To summarize, the present paper has introduced a formulation of two-phase immiscible displacement in porous media based on general balance laws and constitutive assumptions. The theory does not require capillary pressure functions $P_c(S_{\mathbb{W}})$ or relative permeabilities as input parameters. Instead it predicts such a relation under certain conditions as output.

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