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Macroscopic two-phase flow in porous media

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Abstract

A system of macroscopic equations for two-phase immiscible displacement in porous media is presented. The equations are based on continuum mixture theory. The pairwise character of interfacial energies is explicitly taken into account. The equations incorporate the spatiotemporal variation of interfacial energies and residual saturations. The connection between these equations and relative permeabilities is established, and found to be in qualitative agreement with experiment.

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A great deal of interest has recently been focused on the physics of mono- and multiphase fluid flow in porous media [1–11]. Displacement of one fluid by another inside a porous medium is an unsolved problem of great importance in areas such as filtration [12], ground-water hydrology [13], (nuclear) waste storage [14], catalysis [15], chromatography [16], or hydrocarbon exploration and production [17] to name only a few areas of application.

Multiphase fluid flow in porous media is almost universally treated in practical calculations [18,19] by a well-known set of nonlinear partial differential equations [20–23]. A number of severe problems arise within this formulation [24], and we introduce the reader to these problems in the first three introductory paragraphs. In the traditional formulation a rigid porous medium (such as sedimentary rock $\mathscr{R}$) filled with two immiscible and incompressible fluids (such as water w and oil o), is viewed as a macroscopic mixture of three phases in which each phase is simultaneously present at each macroscopic space–time point $(\boldsymbol{x}, t)$. One stipulates mass conservation

$$\phi \frac{\partial S_w}{\partial t} = -\boldsymbol{\nabla} \cdot \bar{\boldsymbol{v}}_w, \tag{1}$$

$$\phi \frac{\partial S_w}{\partial t} = -\boldsymbol{\nabla} \cdot \bar{\boldsymbol{v}}_o,$$

combined with a generalization of Darcy's law [21,22]

$$\bar{\boldsymbol{v}}_w = -\left[\frac{\boldsymbol{K}k^{\mathrm{r}}_w}{\mu_w}(\boldsymbol{\nabla} P_w - \rho_w \boldsymbol{g})\right], \tag{2}$$

$$\bar{\boldsymbol{v}}_o = -\left[\frac{\boldsymbol{K}k^{\mathrm{r}}_o}{\mu_o}(\boldsymbol{\nabla} P_o - \rho_o \boldsymbol{g})\right]$$

for the macroscopic pressure fields $P_w(\boldsymbol{x}, t)$ and $P_o(\boldsymbol{x}, t)$, the macroscopic (Darcy) velocities $\bar{\boldsymbol{v}}_w(\boldsymbol{x}, t)$ and $\bar{\boldsymbol{v}}_o(\boldsymbol{x}, t)$, and the saturations $S_w(\boldsymbol{x}, t)$, $S_o(\boldsymbol{x}, t) = 1 - S_w(\boldsymbol{x}, t)$, all of which together represent the macroscopic state of the system. Remaining symbols are the fluid parameters such as densities ρ_w, ρ_o and viscosities μ_w, μ_o, the absolute permeability tensor $\boldsymbol{K}$ and porosity ϕ of the porous medium, and the acceleration of gravity $\boldsymbol{g}$ as an external body force. Eqs. (1) and (2) are completed by postulating that the capillary pressure

$$P_{\mathrm{c}}(\boldsymbol{x}, t) = P_o(\boldsymbol{x}, t) - P_w(\boldsymbol{x}, t) = P_{\mathrm{c}}(S_{\mathrm{n}}(\boldsymbol{x}, t)) \tag{3}$$

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and the relative permeabilities $k^{\mathrm{r}}_{\mathscr{W}}(\boldsymbol{x},t)=k^{\mathrm{r}}_{\mathscr{W}}(S_n(\boldsymbol{x},t))$, $k^{\mathrm{r}}_{\mathscr{O}}(\boldsymbol{x},t)=k^{\mathrm{r}}_{\mathscr{O}}(S_n(\boldsymbol{x},t))$ are functions of a normalized effective saturation

$$S_{\mathrm{n}}(\boldsymbol{x},t)=\frac{S_{\mathscr{W}}(\boldsymbol{x},t)-S_{\mathscr{W}\mathrm{i}}}{1-S_{\mathscr{W}\mathrm{i}}-S_{\mathscr{O}\mathrm{r}}}, \tag{4}$$

containing the irreducible water (= wetting phase) saturation $S_{\mathscr{W}\mathrm{i}}$, and the residual oil (= non-wetting phase) saturation $S_{\mathscr{O}\mathrm{r}}$ as two additional parameters. Many if not all numerical simulations of macroscopic two-phase immiscible displacement in porous media are based on these equations or on their special cases such as the Buckley–Leverett equations [18,19,25].

Despite of their practial importance Eqs. (1)–(4) are flawed in at least two respects. Experiments [26–29] have shown that the parameters $S_{\mathscr{W}\mathrm{i}}$ and $S_{\mathscr{O}\mathrm{r}}$ are not constant but depend on flow conditions such as velocity. In addition, the relative permeabilities $k^{\mathrm{r}}_{\mathscr{W}}(S_{\mathrm{n}})$, $k^{\mathrm{r}}_{\mathscr{O}}(S_{\mathrm{n}})$ and the capillary pressure $P_{\mathrm{c}}(S_{\mathrm{n}})$ are assumed to be constitutive functions in the mathematical equations while in reality they are functionals reflecting the well-known hysteresis [20,21,25,30].

Given these problems it becomes clear that an improved theory may be obtained from a macroscopic energy balance that includes the interfacial energies. Lately such an approach was presented in Ref. [24]. One possible problem with the equations in Ref. [24] is that the interfacial energy of a phase pair was assigned equally to both phases. Redistribution of energies during fluid movement is suppressed by this static assignment. It is the main purpose of this paper to propose an energy balance that is obtained from the principles of mixture theory. A first presentation of the resulting equations will be presented below. More details and a full discussion will be given elsewhere [31].

The interfacial energy densities are $\sigma_{\alpha\beta}A_{\alpha\beta}$ $(\alpha,\beta\in\{\mathscr{W},\mathscr{O},\mathscr{R}\})$, where $\sigma_{\alpha\beta}$ is the surface tension between phase α and β, and $A_{\alpha\beta}$ is the specific internal surface area, i.e. the surface area between α and β per unit volume. Although it is clear that these interfacial energy densities govern the wetting behaviour they do not appear in Eqs. (1)–(4). Therefore, the characterization of the macroscopic state by pressures P_α, velocities $\boldsymbol{v}_\alpha$ and saturations S_α is incomplete and must be enlarged by specific internal surface areas $A_{\alpha\beta}$ [32,33].

As a second ingredient, it is important to recognize the crucial difference between connected and disconnected fluid configurations [24,34]. In the present approach each fluid phase is split into a connected (percolating) and a disconnected (nonpercolating) subphase. During an immiscible displacement process a finite fraction of each fluid phase becomes trapped inside the pore space. These droplets remain immobile and fixed through large local capillary forces until they coalesce again with their mobile connected parent phase. Dividing each of the two fluid phases into a percolating (connected) and a trapped (disconnected) subphase makes it possible to describe the dynamics of residual phase saturations. The subdivision results in a total of five phases (four fluid, one solid), that will be indexed as is summarized in Table 1. A subphase is called connected or percolating if each of its interior points can be connected to the boundaries of the sample by a path within the subphase. The disconnected subphase is the complement within the chosen superphase.

Table 1
Overview over the subphases and their indices for immiscible displacement of two fluids inside a rigid porous medium

Phase	Index	Phase	Substance
$\mathscr{W}$	1	Connected (percolating) fluid	Water
	2	Disconnected (trapped) fluid	
$\mathscr{O}$	3	Connected (percolating) fluid	Oil
	4	Disconnected (trapped) fluid	
$\mathscr{R}$	5	Rigid porous solid	Rock minerals

Based on the foregoing the macroscopic state of the two immiscible and incompressible fluids is described by several macroscopic fields. Each phase has a velocity $\boldsymbol{v}_\alpha(\boldsymbol{x},t)$ where the meaning of the index $\alpha=1,2,3,4,5$ is given in Table 1. In addition, each phase occupies a volume fraction $\phi_\alpha(\boldsymbol{x},t)=\phi S_\alpha(\boldsymbol{x},t)$ where ϕ is the porosity and $S_\alpha(\boldsymbol{x},t)$ is the saturation field of phase α. The connected fluid phases are described by the pressure fields $P_1(\boldsymbol{x},t)$ and $P_3(\boldsymbol{x},t)$. The macroscopic pressure fields for the disconnected fluid phases are expected to be discontinuous when obtained by averaging or homogenization techniques, and hence are neglected in our approach. With these preparations the macroscopic mass, momentum and energy balances may be derived from general mixture theory [31,35]. For mass conservation we obtain [31]

$$\rho_{\mathscr{W}}\frac{\partial}{\partial t}\phi_1+\rho_{\mathscr{W}}\nabla\cdot(\phi_1\boldsymbol{v}_1)=M_{\mathscr{W}}, \tag{5}$$

$$\rho_{\mathscr{W}}\frac{\partial}{\partial t}\phi_2=-M_{\mathscr{W}}, \tag{6}$$

$$\rho_{\mathscr{O}}\frac{\partial}{\partial t}\phi_3+\rho_{\mathscr{O}}\nabla\cdot(\phi_3\boldsymbol{v}_3)=M_{\mathscr{O}}, \tag{7}$$

$$\rho_{\mathscr{O}}\frac{\partial}{\partial t}\phi_4=-M_{\mathscr{O}} \tag{8}$$

and for momentum conservation of the connected fluid phases we have

$$\nabla(\phi_1P_1)-\phi_1\rho_{\mathscr{W}}\boldsymbol{g}=\boldsymbol{m}_1-M_{\mathscr{W}}\boldsymbol{v}_1, \tag{9}$$

$$\nabla(\phi_3P_3)-\phi_3\rho_{\mathscr{O}}\boldsymbol{g}=\boldsymbol{m}_3-M_{\mathscr{O}}\boldsymbol{v}_3. \tag{10}$$

In these equations M_{w} is the mass transfer per unit volume from disconnected water (phase 2) into connected water (phase 1), and correspondingly M_{o} for oil. Similarly, $\boldsymbol{m}_\alpha$ is the momentum transfer per unit volume from all phases into phase α. We have assumed throughout that $\boldsymbol{v}_2 = \boldsymbol{v}_4 = \boldsymbol{v}_5 = 0$, i.e. that the disconnected fluid phases and the rock are immobile. In spite of this assumption spatiotemporal variations of the disconnected phases are possible by coalescence with and breakup from the connected parent phase.

Only kinetic and interfacial energies are included into the energy balance. The pairwise character of interfacial energies necessitates special considerations that will be detailed elsewhere [31]. Assuming constant wetting conditions, i.e. $A_{\alpha 5} =$ const. for $\alpha = 1, 2, 3, 4$, we obtain the following equations for the four fluid phases

$$\begin{aligned}\nabla \cdot (\sigma_{wo} A_{13} \boldsymbol{v}_3) &+ \frac{\partial}{\partial t}(\sigma_{wo} A_{13} + \sigma_{wo} A_{14}) \\ &= \tfrac{1}{2} M_{w} \boldsymbol{v}_1^2 + \tfrac{1}{2} M_{o} \boldsymbol{v}_3^2 - \boldsymbol{m}_1 \cdot \boldsymbol{v}_1 - \boldsymbol{m}_3 \cdot \boldsymbol{v}_3 \\ &\quad - \phi_1 P_1 \nabla \cdot \boldsymbol{v}_1 - \phi_3 P_3 \nabla \cdot \boldsymbol{v}_3 + E_1 + E_3, \end{aligned} \tag{11}$$

$$\begin{aligned}\nabla \cdot (\sigma_{wo} A_{23} \boldsymbol{v}_3) &+ \frac{\partial}{\partial t}(\sigma_{wo} A_{23}) \\ &= \tfrac{1}{2} M_{o} \boldsymbol{v}_3^2 - \boldsymbol{m}_3 \cdot \boldsymbol{v}_3 - \phi_3 P_3 \nabla \cdot \boldsymbol{v}_3 + E_3, \end{aligned} \tag{12}$$

$$\begin{aligned}\nabla \cdot (\sigma_{wo} A_{13} \boldsymbol{v}_1) &+ \frac{\partial}{\partial t}(\sigma_{wo} A_{13} + \sigma_{wo} A_{23}) \\ &= \tfrac{1}{2} M_{w} \boldsymbol{v}_1^2 + \tfrac{1}{2} M_{o} \boldsymbol{v}_3^2 - \boldsymbol{m}_1 \cdot \boldsymbol{v}_1 - \boldsymbol{m}_3 \cdot \boldsymbol{v}_3 \\ &\quad - \phi_1 P_1 \nabla \cdot \boldsymbol{v}_1 - \phi_3 P_3 \nabla \cdot \boldsymbol{v}_3 + E_1 + E_3, \end{aligned} \tag{13}$$

$$\begin{aligned}\nabla \cdot (\sigma_{wo} A_{14} \boldsymbol{v}_1) &+ \frac{\partial}{\partial t}(\sigma_{wo} A_{14}) \\ &= \tfrac{1}{2} M_{w} \boldsymbol{v}_1^2 - \boldsymbol{m}_1 \cdot \boldsymbol{v}_1 - \phi_1 P_1 \nabla \cdot \boldsymbol{v}_1 + E_1, \end{aligned} \tag{14}$$

where E_α denotes the energy transfer per unit volume from all phases into phase α and we assumed $E_2 = E_4 = E_5 = 0$. These equations have to be completed with a specification of the mass, energy and momentum transfer.

$$M_\alpha = M_\alpha(\phi_1, \phi_2, \phi_3, \phi_4, P_1, P_3, \boldsymbol{v}_1, \boldsymbol{v}_3, A_{13}, A_{14}, A_{23}), \tag{15}$$

$$\boldsymbol{m}_\alpha = \boldsymbol{m}_\alpha(\phi_1, \phi_2, \phi_3, \phi_4, P_1, P_3, \boldsymbol{v}_1, \boldsymbol{v}_3, A_{13}, A_{14}, A_{23}), \tag{16}$$

$$E_\alpha = E_\alpha(\phi_1, \phi_2, \phi_3, \phi_4, P_1, P_3, \boldsymbol{v}_1, \boldsymbol{v}_3, A_{13}, A_{14}, A_{23}), \tag{17}$$

as functions of the other variables. This completes the general equations for two-phase flow with immobile disconnected fluid phases. Of course, our approach can be generalized to the case where the disconnected fluids are mobile.

We now proceed to specify a simple model for the mass, energy and momentum transfer. The momentum transfer is traditionally chosen as a simple generalization of Stokes drag [18]

$$\begin{aligned}\boldsymbol{m}_1 &= P_1 \nabla \phi_1 + \sum_\beta R_{1\beta}(\boldsymbol{v}_\beta - \boldsymbol{v}_1) \\ &= P_1 \nabla_{\phi_1} + R_{13} \boldsymbol{v}_3 - (R_{13} + R_{14} + R_{15}) \boldsymbol{v}_1, \end{aligned} \tag{18}$$

$$\begin{aligned}\boldsymbol{m}_3 &= P_3 \nabla \phi_3 + \sum_\beta R_{3\beta}(\boldsymbol{v}_\beta - \boldsymbol{v}_3) \\ &= P_3 \nabla_{\phi_3} + R_{31} \boldsymbol{v}_1 - (R_{31} + R_{32} + R_{35}) \boldsymbol{v}_3, \end{aligned} \tag{19}$$

where the local resistivities $R_{\alpha\beta}$ are in general tensorial. Next, we assume that locally each connected fluid phase flows without disturbing the other fluid (see Ref. [22, p. 335]). As usual we neglect viscous coupling terms, $R_{13}\boldsymbol{v}_3 \ll (R_{13} + R_{14} + R_{15})\boldsymbol{v}_1$ and $R_{31}\boldsymbol{v}_1 \ll (R_{31} + R_{32} + R_{35})\boldsymbol{v}_3$. We approximate the resistivities of each connected fluid phase as in single-phase flow by an expression of Carman–Kozeny (or hydraulic radius) type [22]

$$R_{13} + R_{14} + R_{15} \approx \frac{A_1^2 \boldsymbol{\tau}_1 \mu_{w}}{\phi_1}, \tag{20}$$

$$R_{31} + R_{32} + R_{35} \approx \frac{A_3^2 \boldsymbol{\tau}_3 \mu_{o}}{\phi_3}. \tag{21}$$

Here $\boldsymbol{\tau}_\alpha$ is the tortuosity (tensor) of phase α and $A_\alpha = \sum_\beta A_{\alpha\beta}$ is its specific internal surface area. The energy transfer has to be consistent with the momentum transfer and we assume

$$E_1 = R_{13} \boldsymbol{v}_3 \cdot \boldsymbol{v}_1 - (R_{13} + R_{14} + R_{15}) \boldsymbol{v}_1^2, \tag{22}$$

$$E_3 = R_{31} \boldsymbol{v}_1 \cdot \boldsymbol{v}_3 - (R_{31} + R_{32} + R_{35}) \boldsymbol{v}_3^2, \tag{23}$$

thereby neglecting interfacial energy transfer.

The mass transfer M_α is a new ingredient in our present formulation whose dependence on other variables has to be taken from experiment. Eqs. (6) and (8) suggest to identify ϕ_2 and ϕ_4 with residual saturations from a desaturation experiment. This idea results in an Ansatz

$$M_{w} = -\rho_{w} \phi \frac{\partial}{\partial t} S_{w\mathrm{i}}(|\boldsymbol{v}_3|), \tag{24}$$

$$M_{o} = -\rho_{o} \phi \frac{\partial}{\partial t} S_{o\mathrm{r}}(|\boldsymbol{v}_1|), \tag{25}$$

where the functions $S_{w\mathrm{i}}(|\boldsymbol{v}_3|), S_{o\mathrm{r}}(|\boldsymbol{v}_1|)$ are the capillary desaturation curves taken from experiment [26–29]. A simple analytical model for such capillary desaturation curves is

$$S_{\mathscr{o}\mathrm{r}}(|\boldsymbol{v}_1|) = S^*_{\mathscr{o}\mathrm{r}}\exp\left(-\frac{C_{\mathscr{w}}\mu_{\mathscr{w}}^2|\boldsymbol{v}_1|^2}{\sigma^2_{\mathscr{w}\mathscr{o}}}\right), \tag{26}$$

$$S_{\mathscr{w}\mathrm{i}}(|\boldsymbol{v}_3|) = S^*_{\mathscr{w}\mathrm{i}}\exp\left(-\frac{C_{\mathscr{o}}\mu_{\mathscr{o}}^2|\boldsymbol{v}_3|^2}{\sigma^2_{\mathscr{w}\mathscr{o}}}\right) \tag{27}$$

with constants $C_{\mathscr{w}}, C_{\mathscr{o}}$ controlling the decay from the "plateau values" $S^*_{\mathscr{w}\mathrm{i}}, S^*_{\mathscr{o}\mathrm{r}}$.

Having specified a concrete model for the mass and momentum transfer Eqs. (15) and (16) it is interesting to compare such a model to the traditional formulation given in Eqs. (1)–(4). Inserting the model assumptions (18–25) into Eqs. (9) and (10) yields for the macroscopic (Darcy) velocities

$$\begin{aligned}\bar{\boldsymbol{v}}_1 &= \boldsymbol{v}_1\phi_1\\ &= \left(\rho_{\mathscr{w}}\phi\frac{\partial}{\partial t}S_{\mathscr{w}\mathrm{i}}(|\boldsymbol{v}_3|) - \frac{A_1^2\boldsymbol{\tau}_1\mu_{\mathscr{w}}}{\phi_1}\right)^{-1}\phi_1^2(\boldsymbol{\nabla}P_1 - \rho_{\mathscr{w}}\boldsymbol{g}),\end{aligned} \tag{28}$$

$$\begin{aligned}\bar{\boldsymbol{v}}_3 &= \boldsymbol{v}_3\phi_3\\ &= \left(\rho_{\mathscr{o}}\phi\frac{\partial}{\partial t}S_{\mathscr{o}\mathrm{r}}(|\boldsymbol{v}_1|) - \frac{A_3^2\boldsymbol{\tau}_3\mu_{\mathscr{o}}}{\phi_3}\right)^{-1}\phi_3^2(\boldsymbol{\nabla}P_3 - \rho_{\mathscr{o}}\boldsymbol{g}).\end{aligned} \tag{29}$$

These equations are analogous to the generalized Darcy law (2) in the traditional formulation. If the velocities $\boldsymbol{v}_1, \boldsymbol{v}_3$ are very small or very large, or if the velocities are independent of time, the first term in the first bracket vanishes. It is now possible to identify relative permeabilities as

$$k^{\mathrm{r}}_{\mathscr{w}} = \frac{\phi_1^3}{kA_1^2\boldsymbol{\tau}_1} = (S_{\mathscr{w}} - S^*_{\mathscr{w}\mathrm{i}})^3\frac{A^2\boldsymbol{\tau}}{A_1^2\boldsymbol{\tau}_1}, \tag{30}$$

$$k^{\mathrm{r}}_{\mathscr{o}} = \frac{\phi_3^3}{kA_3^2\boldsymbol{\tau}_3} = (1 - S^*_{\mathscr{o}\mathrm{r}} - S_{\mathscr{w}})^3\frac{A^2\boldsymbol{\tau}}{A_3^2\boldsymbol{\tau}_3}, \tag{31}$$

where k denotes the absolute permeability of the medium. In the second equality the Carman–Kozeny expression $k = \phi^3/(A^2\boldsymbol{\tau})$ for absolute permeability has been inserted, where A is the specific internal surface and $\boldsymbol{\tau}$ is the tortuosity of the porous medium.

Several comments are in order here because Eqs. (30) and (31) resemble the relative permeabilities of traditional theory. Firstly, they are Brooks–Corey power-law form with cubic exponent as often assumed in theory and experiment [20]. This form appears here as a consequence of Eqs. (20) and (21) which are analogous to expressions for single-phase flow. Secondly, $k^{\mathrm{r}}_{\mathscr{w}}$ and $k^{\mathrm{r}}_{\mathscr{o}}$ show the correct dependence on wettability. If the medium changes from water wet to oil wet with all other quantities held constant then this means in the present formulation that $A_{35} + A_{45}$ increase at the expense of $A_{15} + A_{25}$. Because $A_1 = A_{13} + A_{14} + A_{15}$ and $A_3 = A_{13} + A_{23} + A_{35}$ this implies that A_1 will decrease while A_3 increases. Hence $k^{\mathrm{r}}_{\mathscr{w}}$ increases while $k^{\mathrm{r}}_{\mathscr{o}}$ must decrease, as observed in experiment [22]. Similarly, one expects that during imbibition in a water wet medium the specific surface A_3 will be larger than during drainage because of trapped oil. Hence, one expects that $k^{\mathrm{r}}_{\mathscr{o}}$ for drainage should be larger than for imbibition if all other factors are kept constant.

We pause here to warn the reader that Eqs. (30) and (31) do not hold generally in our approach but only under the special assumptions stated above. During a displacement the specific surfaces themselves change, and hence the relative permeabilities are process and state dependent. We also emphasize that in our approach even the generalization of Darcy's law (28) and (29) is not always valid. If a generalized Darcy's law is valid it contains viscous coupling terms not present in the traditional assumption (2). Unfortunately the approach in its present form does not yield information on capillary pressure functions in gravitational equilibrium. This indicates that the constitutive modeling needs to be further improved, and work in this direction is in progress [31].

In summary, we have presented a formulation of two-phase flow through porous media that replaces traditional concepts such as relative permeabilities with dynamical equations. The approach is based on the physical principles and balance laws of macroscopic continuum mechanics. The distinction between connected and disconnected fluid phases is its main new ingredient. We find that under constitutive assumptions analogous to those for single-phase flow the traditional relative permeabilities may reappear, and that in such cases their saturation dependence is similar to experiment.

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