PHYSICA A

Local porosity theory for electrical and hydrodynamical transport through porous media

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The current status of local porosity theory for transport in porous media is briefly reviewed. Local porosity theory provides a simple and general method for the geometric characterization of stochastic geometries with correlated disorder. Combining this geometric characterization with effective medium theory allows for the first time to understand a large variety of electrical and hydrodynamical flow experiments on porous rocks from a single unified theoretical framework. Rather than reproducing or rephrasing the original results the present review attempts instead to place local porosity theory within the context of other current developments in theory and experiment.

1. Introduction

A two-component porous medium may be characterized as a subset G of a d-dimensional Euclidean space which can be decomposed as $G = P \cup M \cup B$ into three mutually disjoint sets P, M and B. The pore space P and the matrix space M are open subsets of $\mathbb{R}^d$ and their common boundary $B = \partial P = \partial M$ will in general be an irregular stochastic manifold.

Many materials with correlated disorder can be idealized as two-component porous media. The particular example of interest in this paper are porous rocks [1,2] such as sandstones, limestones, dolomites or similar geological formations. Experimental investigations of the electrical resistivity [3–12] and specific permeability [7–15] for a given geological formation often find power law behaviour for the electrical formation resistivity factor F as a function of porosity ϕ,

$$F = \frac{\rho_0}{\rho_W} = a_F \phi^{-m} , \tag{1}$$

and similarly for the specific permeability k,

$$k = a_k \phi^b , \tag{2}$$

in the limit $\phi \rightarrow 0$. Eqs. (1) and (2) imply an algebraic relationship of the form

$$k = a_k a_F^{b/m} F^{-b/m} \tag{3}$$

between permeability k and electrical formation resistivity factor F. In these equations ρ_0 denotes the resistivity of a nonshaly rock sample filled with brine, ρ_W is the resistivity of brine, a_k and a_F are empirical constants and the porosity ϕ is defined as $\phi = V_P/V_G$ the ratio of pore volume V_P and sample volume V_G. Eq. (1) is known as Archie's law. The exponent m is called cementation index and its empirical value is often found to be $m \approx 2$ but it may scatter widely between 1 and 3. Archie's law makes two independent statements: Firstly, the conductivity is finite for all nonzero values of ϕ and, secondly, the cementation index assumes nontrivial values.

Dullien [11] defines porous materials as materials for which the permeability k is nonzero. It is clear then that the permeability k is the most important physical property of porous media. The permeability k is defined [11] as the proportionality constant in Darcy's law which establishes a linear relationship between the current density and an applied external pressure gradient driving a viscous flow through the medium. Darcy's law is obtained from the Stokes equation for viscous flow in a porous medium at low Reynolds numbers by applying well known homogenization techniques [16–19]. Relation (2) is similar to Archie's law (1) and the exponent b is often found to have values around 4. The exponent $h = b/m$ in relation (3) on the other hand scatters experimentally between 1.3 and 2.8 [8,9,20,21] but its value is most often found around $h \approx 2$.

Going from dc conductivities and permeabilities to the corresponding frequency dependent transport coefficients the experimental situation becomes very asymmetric. While there exist many measurements of frequency dependent formation factors or dielectric susceptibilities [1,6,22–26] comparatively little work has been done on measuring frequency dependent permeabilities [27]. The dielectric response of clayfree water saturated sandstones shows strong dispersion at low frequencies although neither rock nor water is dispersive in the same frequency range. The values of the dielectric constant at zero frequency for the water–rock composite often exceed those of both constituents. This phenomenon is called dielectric enhancement.

Let me formulate a number of questions raised by these experimental findings before defining the objective of this paper. An obvious question is why there should exist any kind of correlation between transport quantities such as F or k and geometric quantities such as ϕ. After all the porosity ϕ contains no information about pore throats or pore space connectivity. Why should the correlations follow power laws? What is the meaning of the exponents? Are

they characteristic for different geometries? Are they dynamic exponents? Why is there such a large scatter in their numerical values? Is there some form of universality or are there universal classes of porous media? What are the mechanisms for dielectric enhancement?

The objective of this contribution is to give a brief overview of local porosity theory [28–31] which has recently been developed to address some of these questions. The present paper will begin with a brief review of the electrical problem and then continue on to the flow problem. The discussion will remain verbal and qualitative. However, the reader is advised to consult refs. [28,30] for original results whenever the discussion requires it.

2. Dielectric response

Recent theoretical attempts to calculate the dielectric properties of water filled porous rocks can be classified into two main categories: One group [32–41] attempts geometrical modeling of the pore space P, while the other [42–48] develops phenomenological models based on the spectral representation of the complex dielectric function $\epsilon(\omega)$ developed by Fuchs [42] and Bergman [43]. Only the first group allows to correlate geometrical properties of the pore space with its dielectric response. Most works focus either on an explanation of Archie's law or else on dielectric enhancement. In particular, when studying different geometrical modeling approaches and comparing them with experiments it seems that none of them incorporate Archie's law in its double meaning *and* dielectric enhancement simultaneously. In this situation it was the objective of ref. [28] to develop a theoretical framework which would allow a systematic discussion of all aspects of the dielectric response of general porous media.

The main obstacle to achieving this objective is a classical problem in the theory of heterogeneous media [49,50]: The basic problem is to give a quantitative geometric characterization of the pore space. In the literature the geometric characterization is usually based on so-called pore size distributions [10–12,51,52] which are determined from mercury injection experiments. Unfortunately it is well known that the intuitively appealing concept of pore size distributions is mathematically not well defined [10,11] and experimental determinations of pore size distributions have to be interpreted with care [53]. Similar problems arise from network modeling [54–57] where a certain element of subjective judgement remains part of the mapping procedure. A mathematically well defined geometric characterization is given by correlation functions [58–61]. A problem with this approach is that important qualitative geometri-

cal features of the pore space may be buried in higher order correlation functions which are difficult to measure in experiments. Finally, some authors have employed fractal geometry to give a geometric characterization of pore space [51,62]. However, in view of the limited scaling regime such a characterization remains doubtful.

The basic idea of local porosity theory as proposed in ref. [28] is not to try to identify pores, grains, networks or other geometrical objects within the pore space as the fundamental carrier of randomness but the porosity itself. More precisely, any globally well defined geometric observable such as porosity, or specific internal surface area becomes a local geometric observable in local porosity theory.

The details of implementing this idea are described in ref. [28]. The main outcome of the analysis are local porosity distributions $\mu(\phi)$ and local percolation probabilities $\lambda(\phi)$ as two basic quantitative geometric characterizations of general porous media. The geometric characterization using μ and λ fulfills three important requirements: (1) It is generally applicable and mathematically well defined. (2) It is experimentally observable. (3) It can be used to calculate transport coefficients. Ref. [31] discusses the microstructural sensitivity of local porosity distributions. In addition to the above three requirements a useful geometric characterization should be sufficiently sensitive to microstructure without requiring a full specification of the pore space boundary B. It appears from the results of ref. [31] that local porosity distributions contain geometrical information which cannot be obtained from simple two-point correlation functions.

The functions $\mu(\phi)$ and $\lambda(\phi)$ can be employed directly in a simple generalization of standard effective medium schemes [28] for the calculation of transport properties. As a consequence the well-known percolation singularity of self-consistent effective medium theory [63,64] appears also in local porosity theory. The control parameter controlling the underlying percolation transition is given by the interval

$$p = \int\limits_0^1 \mu(\phi)\, \lambda(\phi)\, d\phi . \tag{4}$$

Therefore the scaling laws for the conductivity can arise not only for $p \to p_c$ but also for $\bar{\phi} \to 0$ where $\bar{\phi}$ is the bulk porosity defined as $\bar{\phi} = \int \phi\mu(\phi)\, d\phi$. The control parameter for the underlying percolation transition decouples from the average porosity.

Further analysis along the lines of effective medium theory gives rise to a new understanding of Archie's law. Archie's law can indeed emerge within

local porosity theory in the limit $\bar{\phi} \to 0$. It becomes evident however that Archie's law is not simply a relation between geometric properties and transport properties of porous media. Instead its validity is related also to the diagenetic history of a geological formation. Local porosity theory predicts the frequently observed value $m = 2$ for the cementation index as a mean field result. The large scatter in the experimental values for m reflects the importance of diagenetic processes. It arises in the theory in two ways: as a crossover from mean field exponents to true percolation exponents, and because the behaviour of $\mu(\phi)$ $\lambda(\phi)$ can induce nonuniversal exponents [63,64].

There are three mechanisms for dielectric enhancement within local porosity theory. One results from the divergence of Re $\epsilon(0)$ at the underlying transition, the second arises from the width of $\mu(\phi)$, and the third from the behaviour of $\lambda(\phi)$ for $\phi \to 1$. Thus local porosity theory is the first theory to accommodate both Archie's law and dielectric enhancement for general porous media. Recently a comparison between the new approach and some of the previous theories has been carried out for the particular case of water saturated sintered glass beads [65,66]. The new approach was found to give superior agreement with experiment.

Finally it should be mentioned that local porosity theory makes an interesting prediction for a new experimental effect. It predicts [28] a power law divergence for the real part of the dielectric constant in the high porosity $\bar{\phi} \to 1$ limit under analogous conditions and restrictions as those required for Archie's law.

3. Permeability

The problem of viscous flow and the calculation of dc permeabilities from local porosity distributions was treated in ref. [30]. In addition to the questions raised in the introduction the motivation came from the results obtained for the dielectric problem. The objective in ref. [30] was to apply the results of ref. [28] to the hydrodynamic flow problem using the analogy between dc permeability and dc conductivity.

The analogy between hydrodynamic and electrical flow arises from homogenizing the Stokes equation [18,19,61,67,68]. As a consequence the characterization of local geometries must be expanded to include also the local specific internal surface area S. The specific internal surface area is the ratio between the area of the pore space surface B and the total volume V_G. The expectation value $\bar{S}$ is the average local specific internal surface area. It has dimensions of inverse length and is an important geometric quantity characterizing the pore space geometry.

Unlike the electrical conductivity the permeability depends also on the absolute length scale of the pores that control the fluid transport. Correspondingly an important question is that of characteristic length scales associated with a porous medium. The traditional answer relates the permeability to the so-called hydraulic radius r_h defined as the ratio of pore space volume and pore space surface area [10,11]. The hydraulic radius arises naturally in capillary tube models [10,11]. This theory has been modified by incorporating the formation factor into $k \propto r_h^2/F$ [20,59]. An interesting aspect of the modified approach is that it accounts for the tortuosity of the pore space through F. The occurrence of r_h on the other hand makes it less attractive, because r_h contains contributions from the dead ends, i.e. from regions of pore space which are not dynamically connected.

An alternative answer to the question of length scales was given as $k \propto l_c^2/F$ in refs. [51,59], where the length l_c is related to the breakthrough pressure in a mercury injection experiment. The length l_c is only well defined when the pore space can be modeled through a broad distribution of cylindrical pores on a lattice. The quantity Λ introduced in ref. [70] is related to l_c as shown in ref. [71] and one has $k \propto \Lambda^2/F$. The calculation of Λ requires knowledge of the electric field throughout the pore space and thus the length Λ is of dynamical rather than geometrical origin. It seems then that the problem of identifying a characteristic length scale for general porous media has not yet found a satisfactory answer.

In this situation the results of ref. [30] provide an additional alternative to answer the question of length scales. It is shown in ref. [30] that

$$k \propto \frac{\bar{\phi}^2}{\bar{S}^2 F} \, , \tag{5}$$

and thus the characteristic length is identified as $\bar{\phi}/\bar{S}$. The length contains the local quantity $\bar{S}$, and thus it may be called a "local hydraulic radius". Because it is a local quantity the arguments against hydraulic radius theories no longer apply.

Another important result of ref. [30] is that it provides a possible answer to the questions concerning scaling laws and universality. It was already found in ref. [28] that the cementation index m not only depends on the underlying percolation singularity but also on the diagenetic or consolidation processes. This means that the concept of universality remains meaningful only inasmuch as it concerns the universality of effective medium theory itself. It turns out that the universal regime for the hydrodynamic flow problem is smaller than that for the electric flow problem. Thus the scaling laws for k should show a

lower degree of universality than Archie's law. This expresses a higher microstructural sensitivity of the permeability and is confirmed by experiment.

Having established the importance of consolidation processes for the value of the exponents in the scaling laws (1)–(3) it is necessary to incorporate such consolidation processes into the theoretical treatment. This has been done in ref. [30] using two different classes of consolidation processes. The objective was to ascertain the typical variability of the exponent values. A general requirement for the observation of scaling laws is that the consolidation processes are p_c-approaching [30]. Preliminary results indicate that the process of sintering glass beads is such a p_c-approaching consolidation process [66]. The value $b = 4$ emerges as a mean field result whose range of validity however is markedly restricted as compared to Archie's law. This is confirmed by the increased scatter of b-values in experiment.

A second class of consolidation processes is defined using a particular physical model, the so-called local porosity reduction model [30]. In the local porosity reduction model the consolidation consists in randomly picking a volume element and reducing porosity and specific surface area within it. The model allows to determine the local porosity distribution exactly and the exponents m, b and $h = b/m$ can be correlated with particular changes of the local pore geometries. It is found that the cementation index m depends only upon changes in local porosity while h is independent of it. The exponent h depends only upon changes in local specific surface area. The exponent b contains contributions from both effects.

Within the local porosity reduction model it is predicted that h should vary at most within $1 \leqslant h \leqslant 3$. Values around $h \approx 2$ indicate consolidation processes in which locally the radius of cylindrical pores shrinks. Larger values indicate processes which close cracks or fractures as e.g. under the application of external pressure. Lower values than 2 indicate the filling of spheroidal pores as e.g. when new material is deposited chemically. These theoretical predictions for the range of h and for the numerical values are again found to be confirmed [8,9,20] by experiment.

Acknowledgements

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