Worksheet 11: Root Finding

July 10, 2017

General Remarks

- The deadline for handing in the worksheets is **Monday, July 17th, 2017, 12:00 noon**.
- This is a short worksheet, for which you can achieve a maximum of 6 points.
- To hand in your solutions, send an email to
  - **Michael Kuron** `mkuron@icp.uni-stuttgart.de` **(all groups!)**
- Please try to only hand in a single file that contains your program code for all tasks. If you are asked to answer questions, you should do so in a comment in your code file. If you are asked for graphs or figures, it is sufficient if your code generates them. You may as well hand in a separate PDF document with all your answers, graphs and equations.
- The worksheets are to be solved in groups of two or three people.

Task 11.1: Root Finding (6 points)

- **11.1.1** (3 points) Implement three Python functions that find the root of a function:
  - `newton(f, fp, x0)`, which uses *Newton's* method to find the root of the function `f`, where `fp` is the derivative of the function and `x0` is the initial value of the iteration.
  - `secant(f, x0, x1)`, which uses the *secant* method to find the root of the function `f`, where `x0` and `x1` are the initial values of the iteration.
  - `regula_falsi(f, x0, x1)`, which uses the *regula falsi* method to find the root of the function `f` in the interval [`x0`, `x1`].

  The methods should terminate when they have reached an accuracy of 10^{-16}. The accuracy of a given iteration is the absolute difference to the previous iteration in Newton's method and the absolute difference between the two guesses for the secant method. For the regula falsi method, the accuracy has to be determined as `abs(f(p))`, where `p` is the newly determined point in the current iteration.
- **11.1.2** (1 point) Use the functions to compute the root of the following functions:
  - $f(x) = x^2 - 1$ (initial guess for Newton's method $x_0 = 3$, initial interval for the secant method and regula falsi $[0, 3]$)
  - $g(x) = \cos(x)$ (initial guess for Newton's method $x_0 = 2$, initial interval for the secant method and regula falsi $[0, 2]$)
- **11.1.3** (1 point) Extend the Python functions from the previous task such that they record the reached accuracy in every iteration. Create a plot with logarithmic scale on the y-axis that shows the accuracy of the three methods versus the iteration number for both cases.
- **11.1.4** (1 point) As before, try to use Newton's method to compute the root of $g(x) = \cos(x)$, but this time start with an initial value of $x_0 = 0$. Then, use the secant method to compute the root of $g(x)$, but start the method with initial values of $x_0 = -1$ and $x_1 = 1$. What is happening? Why do the methods fail?