

Exercise Sheet 5
Advanced Quantum Theory
WS 2010/11

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Exercise 1:

(4 points)

Let π be a representation of a C^* -algebra \mathcal{A} on a Hilbert space \mathcal{H} . Show that π is faithful if and only if it satisfies each of the following equivalent conditions

- (i) $\ker \pi = \{0\}$,
- (ii) $\|\pi(a)\| = \|a\|$ for all $a \in \mathcal{A}$,
- (iii) $\pi(a) > 0$ for all $a > 0$.

Exercise 2:

(4 points)

Prove the equivalence criteria for irreducibility given in the lecture as II.3.6:

- (i) \mathcal{M} is irreducible, i.e. $\mathcal{M}' = \{\lambda \mathbb{I}\}$,
- (ii) every non-zero vector is cyclic,
- (iii) there are no invariant, proper, closed subspaces.

Exercise 3:

(3 points)

Let π_w be the GNS-representation of \mathcal{A} on $\mathcal{B}(\mathcal{H})$. Show that $\ker \pi_w = \{a \in \mathcal{A} : w(b^* a^* a b) = 0 \text{ for all } b \in \mathcal{A}\}$.