

**Exercise Sheet 2**  
**Advanced Quantum Theory**  
**WS 2010/11**

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**Exercise 1:** **(4 points)**

Prove the Cauchy-Schwarz inequality

$$|\langle v_1, v_2 \rangle| \leq \|v_1\| \|v_2\|$$

and show that

$$|\langle v_1, v_2 \rangle| = \|v_1\| \|v_2\|$$

if and only if

$$v_1 = z v_2$$

for some  $z \in \mathbb{C}$  (and  $v_i \neq 0$ ).

**Exercise 2:** **(5 points)**

Prove the triangle inequality for the  $L^p$  spaces with  $p \geq 1$ .

**Exercise 3:** **(3 points)**

Show that multiplication is continuous (in both factors simultaneously) in a  $C^*$ -algebra.