$$\boxed{\mu = \frac{D}{k_B T}} = \frac{1}{6\pi\eta \cdot a}$$

Langevin Equation

- Stokes law = $F_S = -\gamma \cdot \dot{x}$
- Newton's 2nd law $F_N = m\ddot{x}$

$$m\ddot{x} = -\gamma\dot{x} + F_{ext} + \underset{\text{noise}}{\eta(t)}$$

$F_{ext} = 0$

$$V(t) = V_0 \exp(-\underset{\text{damping term}}{\zeta} \cdot t)$$

- zero mean $\langle \eta(t) \rangle = 0$
- gaussian distributed
- white noise

$$\langle \eta(t) \cdot \eta(t') \rangle = 2\gamma k_B T \, \delta(t - t')$$

overdamped Langevin Eq.

$\ddot{x} = 0$

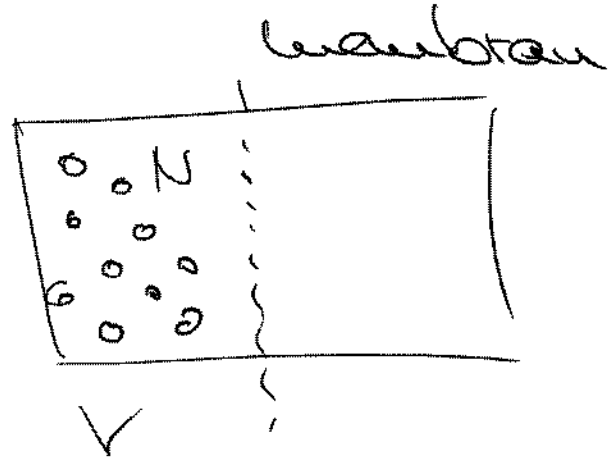
$$\gamma\dot{x} = F_{ext} + \eta(t)$$

Stokes – Einstein – Relation

$$\vec{j} = \vec{j}_D + \vec{j}_{field} \stackrel{!}{=} 0 \qquad \text{equilibr.}$$

$$-D\vec{\nabla}\rho_{eq} + \mu\vec{F}\rho_{eq} = 0$$

ρ_{eq} ?



$$\rho = \frac{N}{V} \qquad P(r) = \rho(r)\cdot k_B T \qquad F = m\cdot g$$

in equilibr. $\rho_e \cdot \vec{F} + \vec{\nabla}\vec{P}(r) = 0$

$$\frac{P(r)\cdot F(r)}{k_B T} = -\vec{\nabla} P(r) \longrightarrow \rho_{eq} \propto P(\vec{r}) \propto \exp\left(\frac{-\vec{F}\cdot\vec{r}}{k_B T}\right)$$

$$\vec{j}_{field} = \vec{v} \cdot \rho$$

mean drift velocity

$$\vec{v} = \mu \vec{F}$$

mobility

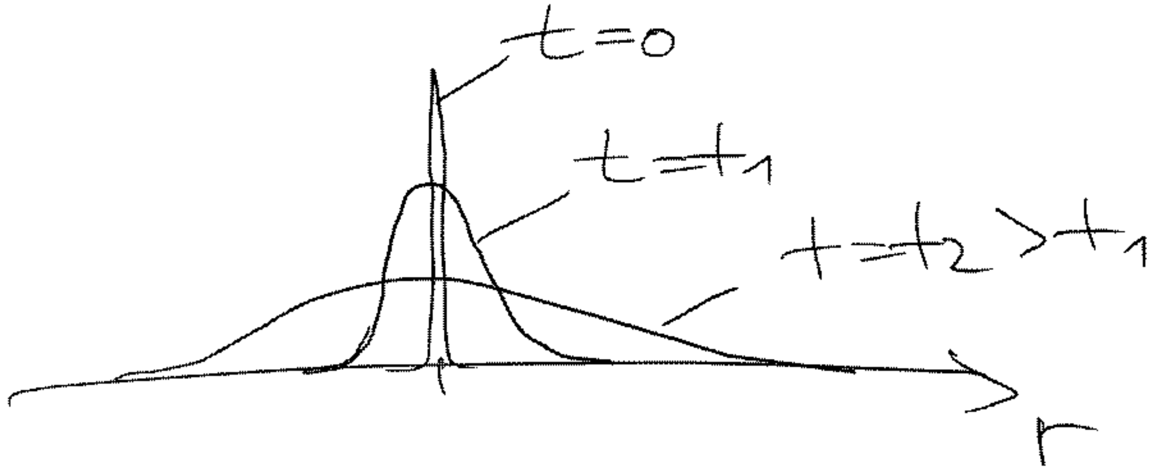
$$m\dot{\vec{v}} + \gamma \vec{v} = \vec{F}$$

friction coefficient

$$\mu = \frac{1}{\gamma} \qquad F = -\underbrace{6\pi\eta a}_{\gamma} v$$

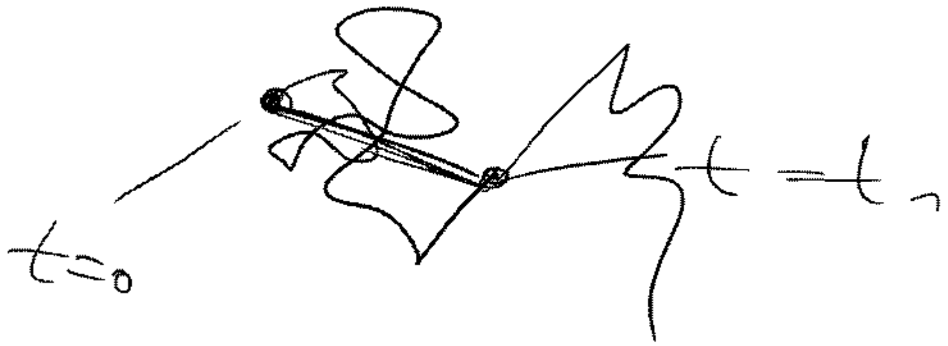
Stationary case $\dot{\vec{v}} = 0$

$$\vec{v} = \vec{F}/\gamma \longrightarrow \vec{\ } = \mu \cdot \vec{F} \cdot \rho$$



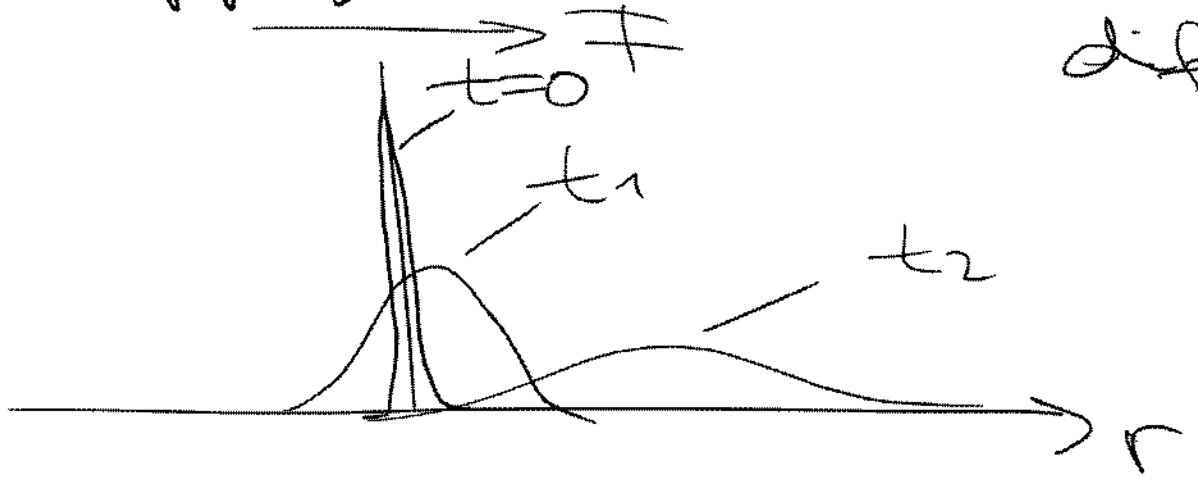
broadening $\hat{=}$ dispersion

mean-square-displacement (MSD)



$$\langle (r(t)-r(0))^2 \rangle = \int d^3r\, \rho(\vec{r},t)\,(r(t)-r(0))^2$$
$$= 6Dt$$

now apply external force



diffusion + drift (lin. Superposition)

continuity equation

$$\frac{\partial \rho(\vec{r},t)}{\partial t} = -\vec{\nabla}\vec{j}(\vec{r},t)$$

→ diffusion equation

$$\frac{\partial \rho(\vec{r},t)}{\partial t} = D \Delta \rho(\vec{r},t)$$

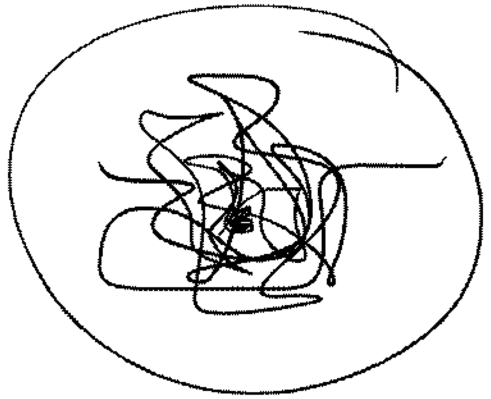
starting condition $\rho(\vec{r},0) = \delta(\vec{r})$

$$\rho(\vec{r},t) = (4\pi D t)^{-3/2} \cdot e^{-\frac{(r(t)-r(0))^2}{4Dt}}$$

gaussian distribution

Historical overview

Solution $\rightarrow$ A. Einstein



trajectorie

random walk

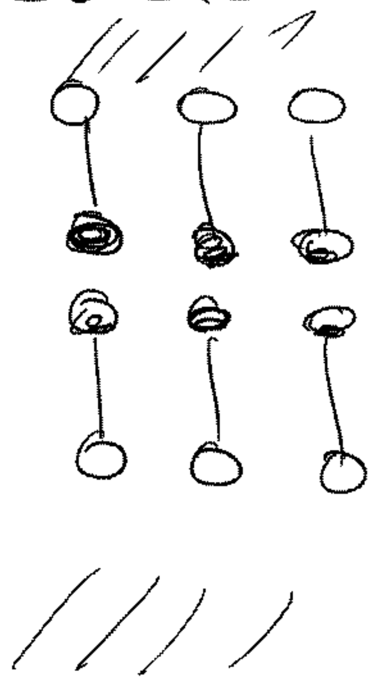
Langevin – theory of Brownian motion

$$\vec{\nabla} \rho(\vec{r},t)$$

$$\vec{j}_D(\vec{r},t) = -D\,\vec{\nabla}\rho(\vec{r},t)$$

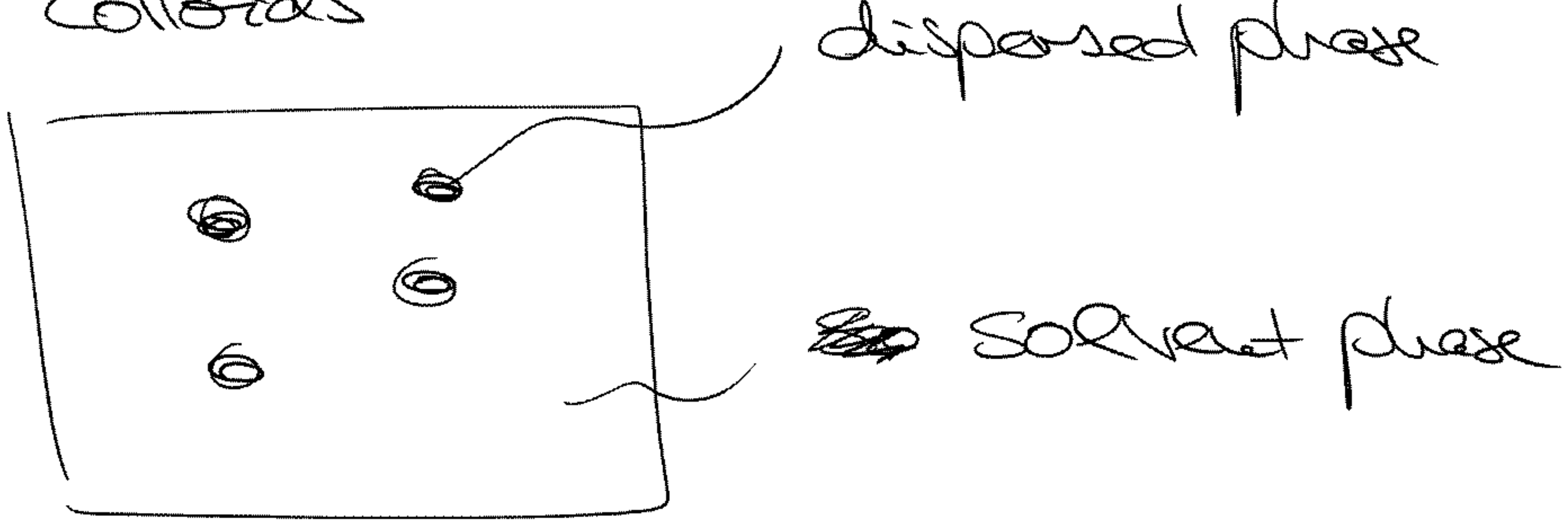
↑ diffusion coefficient

5) vesicles



≙ biolog. cell membrane

→ back to colloids



3) <u>Polymers</u>

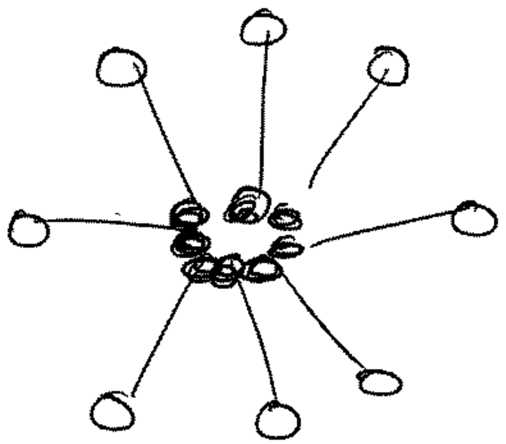
repeating unit consisting of 100 – 10.000 atom

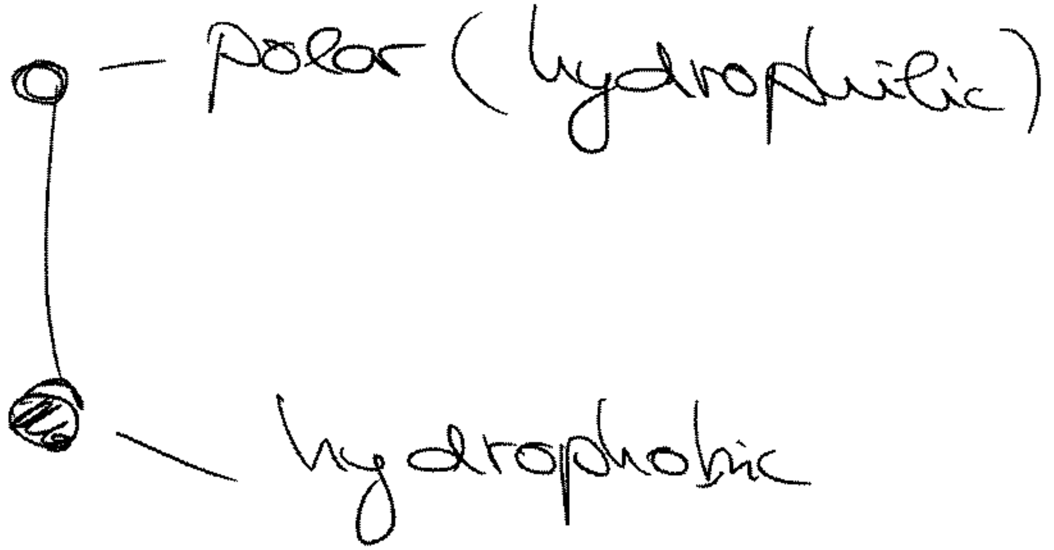
Polyepoxide $\underbrace{\left[(CH_2)_x O\right]_n}_{\text{monomer}}$

4) <u>Micelles</u>

<u>amphiphilic</u> molecule

polar solvent

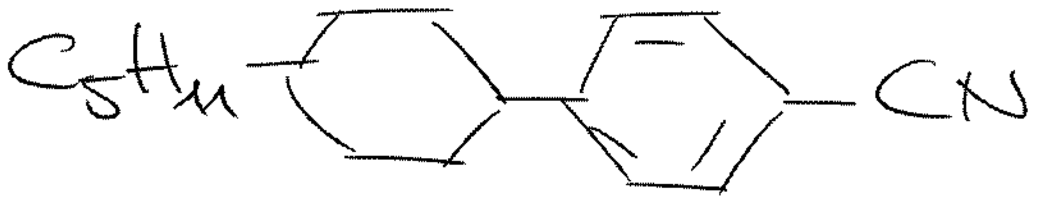


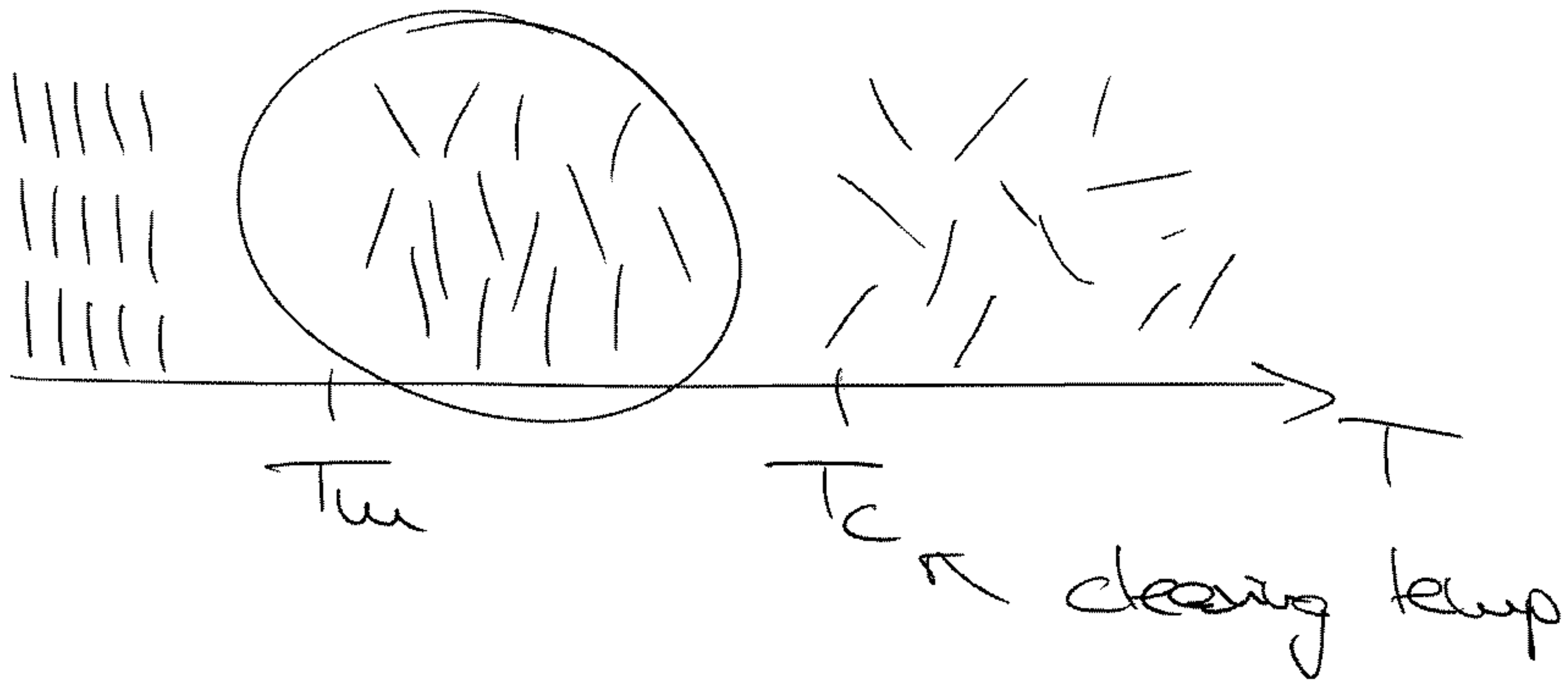


2) liquid crystalls

anisotropic molecüls ~ 10 – 100 atoms

PCH-5





elastic constant: $\dfrac{k_B T}{a^3_{Koll}} \Big/ \dfrac{k_B T}{a^3_{Atom}} \leq 10^{-9}$

→ macromolecules are easily deformable
high compressibility

1) <u>colloidal systems</u>

solid particles suspended in liquid

$a \sim 10nm - 10\mu m$

What is soft matter?

atoms | molecules | colloids, vesicles, micelles, polymers | macroscopic solids

Å | nm | 10n | 100nm | 1μm → size

10ps | 1μs | 1s → relaxation $\tau \propto D \cdot a^2$

- $a_{Koll} / a_{Atom} \geq 10^3$; typ. interaction energy between particles $\sim 1 k_B T \sim \frac{1}{40}$ eV