Tutorial

3: Monte Carlo: The Ising model II

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1 Scaling: Magnetization and Susceptvity

As an example of quantities that shows scaling behavior at the critical temperature

$$KT_c/J = 2/\ln(1+\sqrt(2)) \simeq 2.269185,$$

we will look at the absolute value of the magnetization $\langle |m| \rangle$ and, the susceptibility $\chi^c = L\beta \left\langle m^2 \right\rangle - \langle m \rangle^2$, or – which is equivalent from the pont of view of the scaling behavior – $\chi = L\beta \left\langle m^2 \right\rangle$.

- Modify the code in such a way that configurations are stored every 1 step. Then, with the help of a simple shell script, launch five simulation runs, using box sizes of 4, 8, 16, 32, and 64 units, respectively, like in the following:

```
for i in 4 8 16 32 64 ; do
    ./isim m  2.269185 10000 $i 0 1000 > $i.gpt ;
done
```

This will dump to disk the time series of the magnetization, wich will be analyzed afterwards.

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- Try to write a simple `awk` script to extract from the time series the average values of (the absolute value of) magnetization and its square (Advanced task: modify the analysis script to calculate an estimate for the errors)

- When the analysis tool is complete and the simulation are done, collect in a file the obtained values for $\log_{10}\langle|m|\rangle$ and $\log_{10}\chi$, as functions of $\log_{10}L$ and plot them using `gnuplot`.

- Use `gnuplot`'s `fit` command to fit a straight line to the data. Remembering that

$$\langle|m|\rangle \sim L^{\beta/\nu}, \text{and} \quad \chi \sim L^{\gamma/\nu},$$

  provide an estimate for the two ratios of critical exponent, β/ν and γ/ν.

The resulting scaling curves should look like in Fig:1

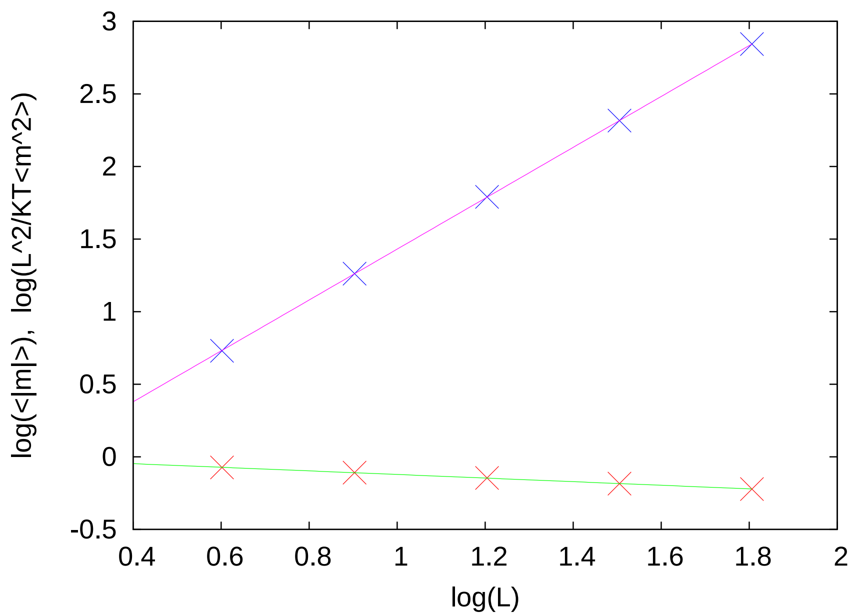

Figure 1: Scaling behaviour of magnetization (lower curve) and susceptibility (upper curve). Points refer to simulation data, while lines are the results of the fit.

2 Derivative of the Binder paramenter

The binder parameter is defined as

$$U_L = 1 - \frac{1}{3}\frac{\langle m^4\rangle}{\langle m^2\rangle^2}.$$

- You should try to evaluate the analytical form of its derivative with respect to β. The function $\frac{dU_L}{d\beta}$ is useful, since it scales like L^ν, being L the size of the lattice, thus allowing to compute directly the exponent ν.

Hint: the resulting function should be

$$\frac{dU_L}{d\beta} = (1 - U_L) \left\{ \frac{\langle m^4 E \rangle}{\langle m^4 \rangle} + \langle E \rangle - 2\frac{\langle m^2 E \rangle}{\langle m^2 \rangle} \right\}$$

- Use the text utility `paste(1)` to generate a file which comprises both the energy and magnetization time series, and modify the previous `awk` script to compute the averages of the observables present in the expression of $\frac{dU_L}{d\beta}$.

- Plot as usual in double logarithmic scale the derivative of the Binder parameter and give an estimate of the critical exponent ν.

- The three critical exponents γ, β, and ν are not independent from each other, and in fact they must satisfy the condition: $2\beta + \gamma = D\nu$, where $D = 2$ is the dimensionality of the problem). Check to which extent this condition is satisfied.