Tutorial

1: Simple and important sampling - Solutions

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1 Introduction

This solution sheet is to be seen as extension of the worksheet distributed in last weeks tutorial. The answers to the Tasks have been provided already during the tutorial. In this document, you find the answers and remarks to the Supplementary tasks.

2 Calculating Π

2.1 Supplementary taks

Standard deviation

Derive a formula to calculate the relative standard deviation of the estimate of Π.

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Assume a probability p to hit in the circle ($p = \frac{d^2/4\Pi}{l^2}$).

The expectation value E is $E = p$ with variance $\sigma^2 = \langle p^2 \rangle - \langle p \rangle^2 = p(1-p)$ (Boolean variables).

After n trials, $E_n = np$ with variance $\sigma_n^2 = np(1-p)$.

Then

$$\frac{\sigma_n}{E_n} = \sqrt{\frac{1-p}{np}}.$$

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As with any counting experiment (and Monte Carlo sampling is essentially counting) the standard deviating scales with the $\frac{1}{\sqrt{n}}$.

Accuracy

Is it a good idea to calculate many decimals of Π using this method?

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No. As the accuracy only improves with 1 over the square root of the number of samples, to increase the accuracy by one digit the computational effort has to be increased by a factor of 100.

3 Numerical integration

3.1 Supplementary tasks

Uniform vs. important sampling

Is a uniform sampling a good choice to integrate such a function?

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It is difficult to correctly sample the fast decaying tails of the Normal distribution with random numbers drawn from a uniform distribution. Therefore a high number of samples has to be taken into account to get a good accuracy.