Exercise Sheet 3
Advanced Quantum Theory
WS 2010/11

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Exercise 1: **(4 points)**

Prove Propositions II.2.9 from the lecture, which is that in a $*$-algebra $\mathscr{A}$

(i) $(a^{-1})^{-1} = a$,

(ii) $(ab)^{-1} = b^{-1}a^{-1}$,

(iii) $(a^*)^{-1} = (a^{-1})^*$,

(iv) the unitary elements form a subgroup of the invertible elements

for all $a, b \in \mathscr{A}$.

Exercise 2: **(3 points)**

A $*$-homomorphism between two C^*-algebras $\mathscr{A}$ and $\mathscr{B}$ is defined as a mapping

$$\begin{aligned} \pi : \mathscr{A} &\to \mathscr{B} \\ a &\mapsto \pi(a) \end{aligned}$$

such that

(i) $\pi(\lambda_1 a_1 + \lambda_2 a_2) = \lambda_1 \pi(a_1) + \lambda_2 \pi(a_2)$,

(ii) $\pi(a_1 a_2) = \pi(a_1)\pi(a_2)$,

(iii) $\pi(a^*) = \pi(a)^*$

for all $a, a_i \in \mathscr{A}$ and $\lambda_i \in \mathbb{C}$.
Show that a $*$-homomorphism is continuous.

Exercise 3: **(3 points)**

Show that

(i) the set of invertible elements of a C^*-algebra is open,

(ii) the set of unitary elements of a C^*-algebra is closed,

(iii) the set of hermitian elements of a C^*-algebra is closed.

(This is Remarks II.1.12 Nr. 3 from the lecture.)