

Advanced Statistical Physics, SS 2017  
Sheet 7

**Problem 1:** (6 points)

Water has its triple point at  $T_3 = 273.2^\circ \text{ K}$ ,  $p_3 = 600 \text{ Pa}$ . The density of ice is  $0.9 \text{ g/cm}^3$  and its heat of fusion is  $80 \text{ cal/g}$ . Recall that one mole of water weighs about  $18 \text{ g}$ .

- Calculate  $\Delta s$ , the change in specific entropy associated with the liquid-solid phase transition near the triple point.
- Calculate  $\Delta v$ , the change of the specific volume associated with the same phase transition.
- Estimate the melting temperature of water under a pressure of 100 atmospheres (1 atmosphere  $\approx 10^5 \text{ Pa}$ ). Assume that  $\Delta v$  and  $\Delta s$  remain approximately constant.
- Does the Clausius-Clapeyron equation explain why ice-skating works?

**Problem 2:** (4 points)

Consider spheres in  $n$  dimensions, where  $n \gg 1$ . Let  $V_n(R)$  be the volume of an  $n$  dimensional sphere of radius  $R$  and  $S_n(R)$  its surface area.

- Explain why  $S_n = \frac{\partial V_n}{\partial R}$ . (*Hint: Think about doing a volume integral in radial coordinates.*)
- Calculate the dimensionless number  $RS_n(R)/V_n(R)$ . What happens to this number for large  $n$ ? Why could this be important?
- Calculate the volume and surface area of a hypersphere. (That is, assume  $V_n(R) = \alpha_n R^n$ , and find  $\alpha_n$ .) (*Hint: One possible method: calculate  $\int e^{-x^2} d^n r$  in two different ways and equate the answer. You can use a different method if you know one.*)

*Deliver your hand-written solutions at the beginning of the lecture on Friday, June 2nd.*