Exercise Sheet 10

Advanced Quantum Theory

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Exercise 1: **(4 points)**

Use the graph $\Gamma(a)$ to show that the inverse a^{-1} of a self-adjoint operator a is self-adjoint, whenever it exists.

Exercise 2: **(3 points)**

Prove the statement II.5.7 Nr.5 .

Exercise 3: **(8 points)**

Repetition:

(a) Plane waves

$$\psi_k(x,t) = \exp\left[i(kx - \omega(k)t)\right]$$

solve the time dependent Schrödinger equation with vanishing potential ($V = 0$). Find the dispersion relation $\omega(k)$.

(b) Due to linearity the general solution is a superposition

$$\psi(x,t) = \int_{-\infty}^{\infty} \phi(k) \exp[i(kx - \omega(k)t)]dk$$

with spectral function $\phi(k)$.

If a free particle of momentum $p_0 = \hbar k_0$ located at $x = 0$ at time $t = 0$ is described by the Gaussian wave packet

$$\psi(x,0) = N \exp(\frac{-x^2}{2a^2} + ik_0 x)$$

with paramters a, k_0 then find the normalization factor N and the spectral function $\phi(k)$

(c) Use $\phi(k)$ to obtain $\psi(x,t)$.

Disscuss the time evolution of the probability density $\rho(x,t) = |\psi(x,t)|^2$. What time does it take to double the width of the wave packet, when the particle is an electron and

$$m = 9.11 \times 10^{-31} \text{ kg}$$

$$a = 0.5 \times 10^{-10} \text{ m}$$

$$\hbar = 1.05 \times 10^{-34} \text{ J s}$$

(The width is the value of x at which $\rho(x)$ is half its maximum)