

# Worksheet 1: Python and NumPy

April 19, 2017

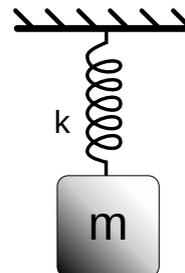
## General Remarks

- The deadline for handing in the worksheets is **Tuesday, May 2nd, 2017, 10:00 a.m.**
- For this worksheet, you can achieve a maximum of 10 points.
- To hand in your solutions, send an email to your tutor:
  - Johannes Zeman zeman@icp.uni-stuttgart.de (Tue 15:45–17:15)
  - Michael Kuron mkuron@icp.uni-stuttgart.de (Wed 15:45–17:15)
  - Kai Szuttor kai@icp.uni-stuttgart.de (Thu 14:00–15:30)
- Attach all required files to the e-mail. If asked to write a program, attach the *source code* of the program. If asked for a text, send it as a PDF file or in plain text format.
- The worksheets are to be solved in groups of two or three people. We will not accept hand-in exercises that only have a single name on it.

## Task 1.1: Spring Pendulum (5 points)

Copy the Python program `pendulum.py` from the home page to your home directory.

The program simulates a spring pendulum, *i.e.* a mass ( $m = 1$ ) that is coupled to a harmonic spring (spring constant  $k = 1$ ) as shown in the figure. Initially, the mass is displaced from the equilibrium position ( $x_0 = 0.3$ ) and at rest ( $v_0 = 0$ ). The force acting on the mass is  $F = -kx$ , the potential energy is  $E^{\text{pot}} = \frac{1}{2}kx^2$ , and the kinetic energy  $E^{\text{kin}} = \frac{1}{2}mv^2$ . To simulate the pendulum, a time step of  $dt = 0.1$  is used and the simulation runs until  $t_{\text{max}} = 10$ . Possibly occurring frictional forces are neglected. At the end, the program plots the position  $x$ , the total energy  $E^{\text{tot}} = E^{\text{kin}} + E^{\text{pot}}$ , and the energy components over time.



- **1.1.1** (2 points) Extend the program such that it simulates two pendulums, one with  $k = 1.0$  and  $x_0 = 0.3$ , the other with  $k = 0.7$  and  $x_0 = 0.1$ . Let the program create a figure that shows four subplots:
  - the positions  $x$  of both pendulums vs. time
  - the total energy  $E^{\text{tot}}$  of both pendulums vs. time
  - the kinetic energy  $E^{\text{kin}}$  of both pendulums vs. time
  - the potential energy  $E^{\text{pot}}$  of both pendulums vs. time

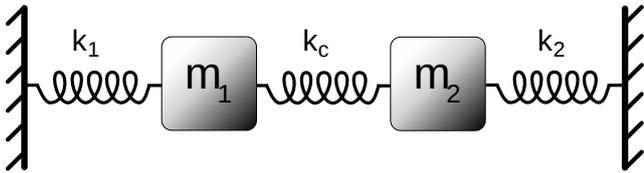
**Hint:** Use `pydoc matplotlib.pyplot.subplot` to get help on how to create the sub plots, or have a look at the matplotlib gallery at <http://matplotlib.org/gallery.html>.

- **1.1.2** (3 points) Change the algorithm of a time step such that it first computes the new positions from the old velocities, and only then the new velocities (the forces are still to be computed at the beginning of the method). Let the program create a plot of the total energy of

the system both for the original algorithm (the *Symplectic Euler algorithm*) as well as for the modified algorithm (the *Euler forward algorithm*) and perform the simulation for  $t_{\max} = 30$ .

How does the behavior of the pendulums differ between the two algorithms? Which of both algorithms would you generally prefer in a simulation?

### Task 1.2: Coupled Spring Pendulum (5 points)



Now, the program from the previous task should be extended to simulate a *coupled* spring pendulum. A coupled spring pendulum consists of two masses  $m_1 = m_2 = 1$  which are fixed between walls with three springs (spring constants  $k_1 = 1$ ,  $k_2 = 0.7$ ,  $k_c = 0.2$ ) as in the figure above<sup>1</sup>. Gravitation and frictional forces are assumed to be negligible.

Initially, the masses are at rest ( $v_1(0) = v_2(0) = 0$ ) and both are displaced from their equilibrium positions ( $x_1(0) = 0.3$  and  $x_2(0) = 0.2$ ). The forces acting on both masses are  $F_1 = -k_1 x_1 - k_c(x_1 - x_2)$  and  $F_2 = -k_2 x_2 - k_c(x_2 - x_1)$ . The “energy of pendulum  $i$ ” is  $E_i = \frac{1}{2} (m_i v_i^2 + k_i x_i^2)$ , and the energy stored in the coupling spring is  $E_c = \frac{1}{2} k_c (x_1 - x_2)^2$ . The total energy of the system is thus given as  $E^{\text{tot}} = E_1 + E_2 + E_c$ .

Extend the program to simulate the coupled spring pendulum for  $t_{\max} = 30$  time units. Let the program create a figure with three subplots showing

- the positions  $x_i$  of both masses vs. time
- the energies of both pendulums  $E_i$  as well as the energy stored in the coupling spring  $E_c$  vs. time
- the total energy  $E^{\text{tot}}$  of the system vs. time

**Hint** The length of the springs is not relevant as long as the equilibrium length is larger than the maximum displacement.

<sup>1</sup>Sketch by jim.belk, CC-BY-SA 3.0, [http://en.wikipedia.org/wiki/File:Coupled\\_Harmonic\\_Oscillator.svg](http://en.wikipedia.org/wiki/File:Coupled_Harmonic_Oscillator.svg)