Physics of Soft and Biological Matter II: Problem Set 2

Owen A. Hickey

April 23, 2014

Problem 1 *Single-File Diffusion, 5 points*

Read the paper Straightforward derivation of the long time limit of the mean-square displacement in one-dimensional diffusion

`http://journals.aps.org/pra/pdf/10.1103/PhysRevA.45.4173`

a) Explain in words where equation 4 and equation 10 come from.
b) Show that by inserting equation 4 into equation 10 one can derive equation 11.
c) How is equation 11 different from a regular one-dimensional random walk of step length l and time step τ where $D = l^2/2\tau$. In particular discuss the exponent of t and why the diffusion coefficient depends on the density of vacancies.

. . . . . .

Problem 2 *Poiseuille Flow in a Cylinder, 5 points*

a) Derive the velocity profile for a pressure driven (Poiseuille) flow in an infinite cylinder of radius R with a fluid of dynamic viscosity η using the Stokes equation:

$$\eta \nabla^2 \vec{u} - \nabla p + \vec{f} = 0 \tag{1}$$

for a given pressure gradient ∇p in the absence of an external force $\vec{f}$.
b) Derive the total fluid flux Q through the cylinder.
c) Calculate the force acting on the cylinder per unit length L using the definition of viscosity. Derive the same result making use of the definition of pressure.

. . . . . .

Problem 3 *Blood Flow, 5 points*

Do problem 5.5 from Nelson, Nelson

`http://www.math.colostate.edu/\~yzhou/course/math676\_spring2013/biophys_Nelson.pdf`

. . . . . .

Problem 4 *Rotational Flow Between Two Cylinders, 5 points*

Consider a cylind of radius R_1 spinning with angular velocity ω inside a larger cylinder of radius R_2. Both cylinder have length L which is much greater than R_2 and there is a fluid between the two cylinders of dynamic viscosity η. Assume there is no external force or pressure drop in the system.
a) Derive the velocity profile between the two cylinders again using the Stokes equation.
b) Calculate the force acting on the two cylinders

. . . . . .

Problem 5 *Couette Flow Over a Polymer Layer, 5 points*

Consider two plates of crosssection A with one plate at $y = 0$ with velocity $v = 0$ and one plate at $y = H_0$ with velocity $v = v_0$. Between the plates is a fluid of dynamic viscosity η. In addition there is an adsorbed polymer layer of height H_p above the lower plate which is free to move with the fluid but has a dynamic viscosity η_p.
a) Given that the fluid velocity and the viscous stress must be continuous derive the velocity profile between the two plates. b) Calculate the force needed to drive the upper plate.

. . . . . .