Problem Sheet 1

for the lecture "Statistical Physics", Master course "Computational Science", year 2007/08

due date: Tuesday, October 23, 2007

Problem 1 1.5 points

You and the bank play the following game: You flip n coins: If t of them come up "heads", you receive 2^t euros.

1. Each time you have to buy a ticket to play the game. What is the fair price of the ticket?
2. Calculate the standard deviation of the variable 2^t.

Problem 2 1.5 points

A diagnostic test for a certain disease has the sensitivity 0.80 and the specificity 0.95. The prevalence of the disease in the population is 0.01 (i.e. one percent of the individuals is suffering from the disease). We shall define S^+ as the event that an individual is suffering from the disease. Correspondingly, the event S^- implies that the individual is healthy. Furthermore, let T^+ and T^- represent the two possible outcomes of the test. In symbols, we may express our prior knowledge as $\mathrm{P}(S^+) = 0.01$, $\mathrm{P}(S^-) = 1 - 0.01 = 0.99$, $\mathrm{P}(T^+/S^+) = 0.80$, $\mathrm{P}(T^-/S^+) = 1 - 0.80 = 0.20$, $\mathrm{P}(T^-/S^-) = 0.95$ and $\mathrm{P}(T^+/S^-) = 1 - 0.95 = 0.05$.

1. What is the probability that a person being diagnosed as diseased is actually diseased?
2. What is the probability of a wrong diagnose?
3. What is the probability of a wrong diagnose if we just declare all individuals for healthy?

Problem 3 1.5 points

Effectiveness of contraceptive methods is measured as the average number of pregnancies per year among 100 couples using the method. Assume that the effectiveness of a method is 0.05 or 5% (i.e. the average number of pregnancies among 100 couples using the method is 5 per year).

1. What is the probability of at least one undesired pregnancy for a couple using the method for 5, 10 and 15 years?
2. Assume instead that the effectiveness is 0.01. What is then the probability of at least one undesired pregnancy for a couple using the method for 5, 10 or 15 years?
3. Choose one of the many existing methods available nowadays (look info for instance in internet) and compute the probability of undesired pregnancy for 5, 10 and 15 years.

Problem 4 1.5 points

Use Stirlings formula to show that

$$\binom{2m}{m} \sim \frac{4^m}{\sqrt{\pi m}}. \tag{1}$$

Problem 5 1.5 points

Suppose we toss a needle of unit length at random onto the plane with equally spaced parallel lines a unit distance apart. Show that the probability that the needle lands on a line is $2/\pi$. Now, check it experimentally: do the experiment at least 100 times, estimate the value of π you get after 10, 30, 50, and 100 trials, does the estimate converge towards the real value of π?

Problem 6 2.5 points

Let's assume we have an ideal gas of N molecules in a box of volume V.

1. What is the probability p of finding one molecule in a volume $V_1 < V$.
2. What is the probability $W(n, V_1, N)$ of finding n of the total N molecules inside the volume V_1.
3. What is the expected value of molecules $\langle n \rangle_{V_1}$ inside a volume V_1. Does this expected value depend on the shape of the box?
4. What is the variance of n, i.e. the mean square deviation (aka σ_n^2)?
5. Consider a sample of gas containing $2.7 \cdot 10^{25}$ molecules in a volume of $1m^3$ of air. Let's assume a volume $V_1 = 100$ litres. Show that the relative deviation respect $\langle n \rangle$, i.e. $\frac{\sqrt{\sigma_n^2}}{\langle n \rangle}$ is extremely small.
6. What conclusions can be derived from the previous point in reference to macroscopic systems?

Hint

In some of these exercises the use of the binomial theorem can be quite helpful,namely

$$(p+q)^N = \sum_{n=0}^{N} \binom{N}{n} p^n q^{N-n} \tag{2}$$