Universität Stuttgart **Prof. Dr. Rudolf Hilfer**
Fakultät 8, Fachbereich Physik **Tillmann Kleiner**

Advanced Statistical Physics, SS 2017
Sheet 5

Problem 1: **(3 points)**

Show that

$$S(X) = \sup_{\substack{X_1, X_2 \\ X = X_1 + X_2}} S(X_1) + S(X_2), \tag{1}$$

using extensivity and concavity of S. The supremum is taken over all pairs of states X_1, X_2 such that $X = X_1 + X_2$.

Problem 2: **(5 points)**

Use the properties of the thermodynamic state function $S(U, V, N)$ to show that the thermodynamic state function $U(S, V, N)$ has the following properties.

a) $U(S, V, N)$ is extensive.
b) $U(S, V, N)$ is differentiable and monotonically increasing in S.
c) $U(S, V, N)$ is convex.

Problem 3: **(6 points)**

The thermal and caloric equations of state of an ideal gas are

$$\frac{pV}{T} = NR \quad \text{(thermal)}, \qquad\qquad U = \frac{3}{2}NRT \quad \text{(caloric)}. \tag{2}$$

a) Use this information to find $S(T, V, N)$.
b) Calculate each of the thermodynamic potentials presented in class:

$$S(U, V, N), \quad U(S, V, N), \quad F(T, V, N), \quad H(S, p, N), \quad G(T, p, N) \quad \text{and} \quad \Omega(T, V, \mu)$$

Hint: Give your results in a dimensionless form. For example, let $S_0 = S(V_0, T_0, N_0)$ *and write* S *in the form* $S_0 + f(V/V_0, T/T_0, N/N_0)$ *where* f *is a function with dimensionless arguments and* $f(1, 1, 1) = 0$.

Deliver your hand-written solutions at the beginning of the lecture on Friday, May 19th.