Physics of Soft and Biological Matter II: Problem Set 2

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Problem 1 *MSD of an Active Particle, 10 points*

Download the paper Non-Gaussian behaviour of a self-propelled particle on a substrate

`http://arxiv.org/abs/0906.3418`

a) Derive equation 11 using equation 10 (note that equation 3 is also needed as mentioned in the paper). Obtain equation 12 by a Taylor expansion of equation 11.

b) How do the authors obtain equation 14? Why is the second term zero? Based on the regular Langevin equation, why is the third term $2t/(\beta^2 D)$? Explain the terms in equation 15.

c) Sub equation 17 into equation 14 along with the result for the third summand of the integral $(2t/(\beta^2 D))$ to get equation 18. Hint: The first integral should first be done for t_1 from t_2 to t and multiplied by two since the equation 17 is only valid for $t_1 > t_2$ and the whole time domain needs to be taken into account. Ignoring terms of order less than t in equation 18 derive the diffusion coefficient given in equation 19.

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Problem 2 *The 3 Bead Swimmer, 10 points*

Download the paper Analytic results for the three-sphere swimmer at low Reynolds number

`http://arxiv.org/abs/0711.3700`

a) Explain where equations 1-3 come from. Why do the forces have to add up to zero (equation 5). Derive equation 9 for the simplified case $a = a_1 = a_2 = a_3$ to leading order in a/L.

b) Using equation 9 calculate the displacement for the cycle:
1) L_2 goes from l_0 to l_f during a time t with constant velocity while $L_1 = l_0$ remains constant.
2) L_1 goes from l_0 to l_f during a time t with constant velocity while $L_2 = l_f$ remains constant.
4) L_1 goes from l_f to l_0 during a time t with constant velocity while $L_2 = l_f$ remains constant.
4) L_2 goes from l_f to l_0 during a time t with constant velocity while $L_1 = l_0$ remains constant.
Hint: Write the integral down as a function of dt and convert it to and integral over dL_i using $dL_i/dt = v_i$, where v_i is the velocity with which L_i expands or contracts during the cycle.

c) Again using equation 9 calculate the displacement for the cycle:
1) L_2 goes from l_0 to l_f during a time t with constant velocity while $L_1 = l_0$ remains constant.
2) L_1 goes from l_0 to l_f during a time t with constant velocity while $L_2 = l_f$ remains constant.
4) L_2 goes from l_f to l_0 during a time t with constant velocity while $L_1 = l_f$ remains constant.
4) L_1 goes from l_f to l_0 during a time t with constant velocity while $L_2 = l_0$ remains constant.
Hint: Write the integral down as a function of dt and convert it to and integral over dL_i using $dL_i/dt = v_i$, where v_i is the velocity with which L_i expands or contracts during the cycle.

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