Tutorial

3: Monte Carlo: The Ising model II

Marcello Sega*

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SimBio group, FIAS, Frankfurt

Contents

1 Scaling: Magnetization and Susceptvity

As an example of quantities that show scaling behavior at the critical temperature

$$KT_c/J = 2/\ln(1+\sqrt(2)) \simeq 2.269185,$$

we will look at the absolute value of the magnetization $\langle|m|\rangle$ and, the susceptibility $\chi^c = L\beta\left\langle m^2\right\rangle - \langle m\rangle^2$, or – which is equivalent from the pont of view of the scaling behavior – $\chi = L\beta\left\langle m^2\right\rangle$.

- Modify the code in such a way that configurations are stored every 1 step. Then, with the help of a simple shell script, launch five simulation runs, using box sizes of 4, 8, 16, 32, and 64 units, respectively, like in the following:

```
for i in 4 8 16 32 64 ; do
    ./isim m  2.269185 10000 $i 0 1000 > $i.gpt ;
done
```

This will dump to disk the time series of the magnetization, wich will be analyzed afterwards.

*sega@fias.uni-frankfurt.de

- Write a simple `awk` script to extract from the time series the average values of (the absolute value of) magnetization and its square.
- When the analysis tool is complete and the simulation are done, collect in a file the obtained values for $\log_{10}\langle|m|\rangle$ and $\log_{10}\chi$, as functions of $\log_{10} L$ and plot them using `gnuplot`.
- Use `gnuplot`'s `fit` command to fit a straight line to the data. Remembering that

$$\langle|m|\rangle \sim L^{\beta/\nu}, \text{and} \quad \chi \sim L^{\gamma/\nu},$$

  provide an estimate for the two ratios of critical exponent, β/ν and γ/ν.

The resulting scaling curves should look like in Fig:1

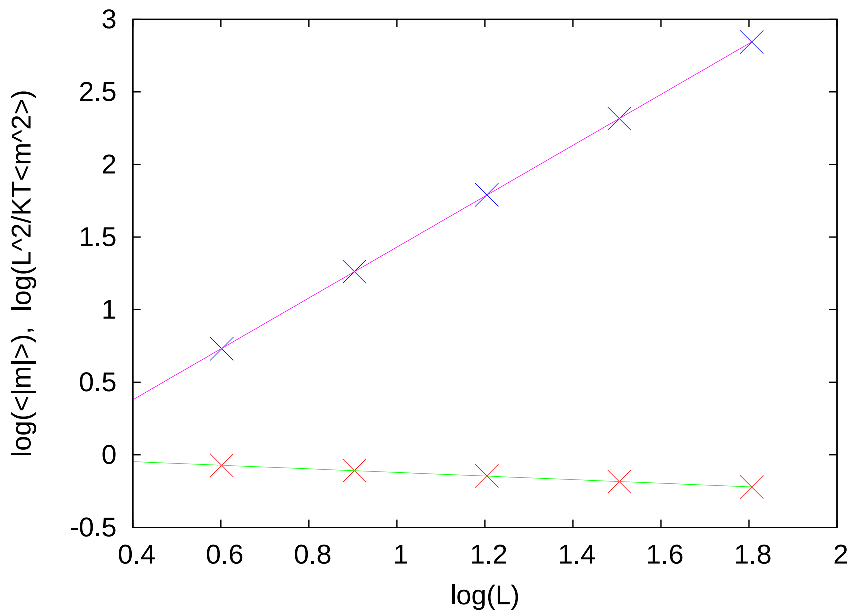

Figure 1: Scaling behaviour of magnetization (lower curve) and susceptibility (upper curve). Points refer to simulation data, while lines are the results of the fit.

1.1 Homework

modify the analysis script to calculate an estimate for the errors on the ratios of the critical exponents.

2 Derivative of the Binder paramenter

The Binder parameter is defined as

$$U_L = 1 - \frac{1}{3}\frac{\langle m^4 \rangle}{\langle m^2 \rangle^2}.$$

2.1 Homework

Compute the analytical form of its derivative with respect to β. The function $\frac{dU_L}{d\beta}$ is useful, since it scales like L^ν, being L the size of the lattice, thus allowing to compute directly the exponent ν. Hint: the resulting function should be

$$\frac{dU_L}{d\beta} = (1 - U_L)\left\{\frac{\langle m^4 E\rangle}{\langle m^4\rangle} + \langle E \rangle - 2\frac{\langle m^2 E\rangle}{\langle m^2\rangle}\right\}$$

2.2 Scaling of the Binder parameter

- Use the text utility `paste(1)` to generate a file which comprises both the energy and magnetization time series, and modify the previous `awk` script to compute the averages of the observables present in the expression of $\frac{dU_L}{d\beta}$.
- Plot as usual in double logarithmic scale the derivative of the Binder parameter and give an estimate of the critical exponent ν.

2.3 Homework

The three critical exponents γ, β, and ν are not independent from each other, and in fact they must satisfy the condition: $2\beta + \gamma = D\nu$, where $D = 2$ is the dimensionality of the problem. Check to which extent this condition is satisfied (give an estimate using error analysis on your datasets)