

Exercise Sheet 11
Advanced Quantum Theory
WS 2010/11

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Exercise 1: **(3 points)**

Verify III.1.3 (iv).

Exercise 2: **(3 points)**

Verify the x -representation of u in III.1.6, Nr. 2.

Exercise 3: **(10 points)**

Repetition:

Consider the anharmonic oscillator with Hamilton operator

$$H = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{m\omega^2}{2} x^2 + k \frac{m^2 \omega^3}{2\hbar} x^4.$$

(a) Rewrite $H(x, p)$ as $H(a, a^+)$ with the creation and annihilation operators

$$a = \sqrt{\frac{m\omega}{2\hbar}} \left(x + \frac{ip}{m\omega} \right)$$
$$a^+ = \sqrt{\frac{m\omega}{2\hbar}} \left(x - \frac{ip}{m\omega} \right)$$

with $[a, a^+] = 1$.

(b) Transform $H(a, a^+)$ into the form

$$H(a, a^+) = Aa^+a + B(a^+a)^2 + C + D(a^+)^2 + D^*a^2 +$$
$$+ F(a^+)^3a + F^*a^+a^3 + G(a^+)^4 + G^*a^4$$

and determine A, B, C, D, F, G .