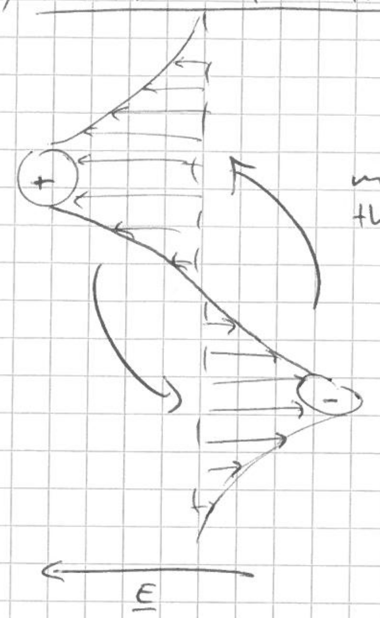
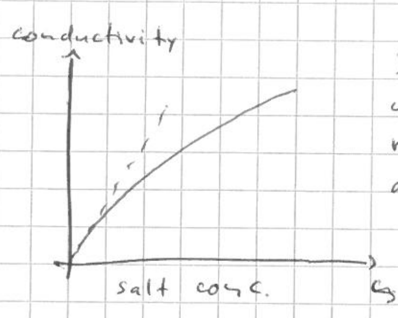
Hydrodynamic interactions



momentum exchange through fluid

Video sedimentation instability



Decrease in cond. of electrolyte rel. to. c_s is partially due to hyd. int.

$Sc = \frac{\eta_{kin}}{D}$ „ratio of momentum transport to mass transport"

$\approx 10^{-2} - 10^{-3}$ for molecular liquids

$\approx 10^{6}$ for µm-sized colloids

Momentum transport is typically much faster than mass transport (particles). Flow can be assumed stationary on small (µm) scales.

Governing equations for fluid flow

Momentum is a conserved quantity. Special relativity dictates that there must be an continuity eq.

stress tensor, density, velocity, external force

$$\underbrace{\nabla \cdot \underline{\underline{\Pi}}}_{\text{momentum flux}} = -\partial_t \underbrace{(\rho \underline{u})}_{\text{momentum density}} + \underline{f}$$

← momentum sources

c.f. $\nabla \cdot \underline{j} = -\partial_t c + R$ (flux, density, sources sinks)

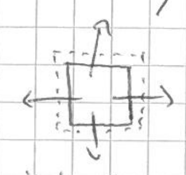
$\underline{\underline{\Pi}}\,\hat{\underline{n}}$ „momentum transported in direction $\hat{\underline{n}}$"

$\underline{j}\cdot\hat{\underline{n}}$ flux in dir. $\hat{\underline{n}}$

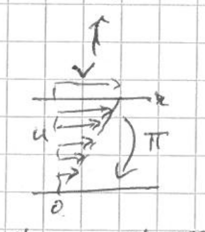
For an incompressible, Newtonian fluid

$$\underline{\underline{\Pi}} = p\,\underline{\underline{1}} - \underbrace{\eta(\nabla \underline{u} + \nabla \underline{u}^T)} + \underline{\rho \underline{u} \otimes \underline{u}}$$

$\underline{u}\otimes\underline{u}$ is projector onto $\underline{u}$ and quadratic in $|\underline{u}|$



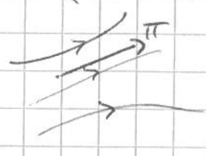
isotropic pressure



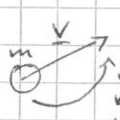
shear stress momentum flux in dir. of shear rate

prefactor η viscosity

e.g. $\eta_{tar} > \eta_{honey} > \eta_{water} > \eta_{air}$



convective momentum flux (in direction of flow)



momentum $p = m \cdot v$ transported over length $v \cdot dt$

Plugging $\underline{\underline{\Pi}}$ into continuity eq., using incompressibility $\nabla \cdot \underline{u} = 0$, $\rho = \text{const}$, yields (after some tensor calculus)

$$\boxed{\begin{gathered} \rho \left[\partial_t \underline{u} + (\underline{u} \cdot \nabla) \underline{u} \right] = -\nabla p + \eta \nabla^2 \underline{u} + \underline{f} \\ \nabla \cdot \underline{u} = 0 \end{gathered}}$$

incompressible Navier-Stokes Eq. for a Newtonian fluid

De-dimensionalization with characteristic length a, velocity v and time τ yields

$$\frac{\rho a^2}{\eta \tau} \partial_t \underline{u}' + Re\, (\underline{u}' \cdot \nabla) \underline{u}' = -\nabla p' + \nabla^2 \underline{u}' + \underline{f}'$$

$$Re = \frac{\rho v a}{\eta}$$ „ratio of convective to viscous momentum transport"

$\approx 10^5$ for macroscopic systems (water at 20°C, 0,1 m sphere falling at 1 m/s)

$\approx 10^{-5}$ for colloidal systems or bacteria ($a = 1\,\mu m$, $v = 10\,\mu m/s$)

Similarity principle: Flows in the same geometry and with the same Re are similar.

Different flow videos

„Small animals experience swimming in water like humans experience swimming in honey or even tar."

In typical soft-matter systems $Re \ll 1$, which allows us to use the simpler Stokes Eq.

$$\boxed{\begin{gathered} \eta \nabla^2 \underline{u} = \nabla p - \underline{f} \\ \nabla \cdot \underline{u} = 0 \end{gathered}}$$ Stokes Eq.

Since this eq. is linear, it has a useful Green's function

$$\underline{f} = \underline{F} \cdot \delta(\underline{r}) \quad \Rightarrow \quad \begin{cases} \underline{u}(\underline{r}) = \frac{\underline{F}}{8\pi\eta} \left(\frac{1}{r} \underline{\underline{1}} + \frac{1}{r^3}\, \underline{r} \otimes \underline{r} \right) \\ p(\underline{r}) = \frac{1}{4\pi r^3}\, \underline{F} \cdot \underline{r} \end{cases}$$ Oseen-Tensor

There are improvements over the Oseen-Tensor, ~~p.a.~~ e.g. the Rotney-Prager-Tensor, which assumes spheres with no-slip BC and incorporates rotations.

This allows one to treat HI as pair-forces. These tensors yield a linear relation between all particle forces and all particle velocities. One can determine the velocities given all conservative forces. ~~by solving a dense linear system of eq. in every time step.~~

→ Stokesian dynamics, Brownian dynamics.