

Worksheet 1: Python and NumPy

April 17, 2013

General Remarks

- Deadline is **Tuesday, 23rd April 2013, 10:00**
- On this worksheet, you can achieve a maximum of 10 points.
- To hand in your solutions, send an email to
 - Olaf (olenz@icp.uni-stuttgart.de; Wednesday, 14:00–15:30)
 - Elena (minina@icp.uni-stuttgart.de; Wednesday, 15:45–17:15)
 - Tobias (richter@icp.uni-stuttgart.de; Friday, 15:45–17:15)
- Attach all required files to the mailing. If asked to write a program, attach the *source code* of the program. If asked for a text, send it as PDF or in the text format. We will *not* accept MS Word files!
- The worksheets are to be solved in groups of two or three people. We will not accept hand-in-exercises that only have a single name on it.
- Die Übungen finden statt im CIP-Pool des Instituts für Computerphysik (ICP) im Pfaffenwaldring 27.

Task 1.1 (4 points): Spring Pendulum

Copy the Python program `pendulum.py` from the home page to your home directory.

The program simulates a spring pendulum, *i.e.* a mass ($m = 1$) that is coupled to a harmonic spring (spring constant $k = 1$), as shows in figure 1. Initially, the mass is displaced from the equilibrium position ($x_0 = 0.3$) and at rest ($v_0 = 0$). The force that acts on the mass is $F = -kx$, the potential energy is $E_{\text{pot}} = \frac{1}{2}kx^2$ and the kinetic energy $E_{\text{kin}} = \frac{1}{2}mv^2$. To simulate the pendulum, it uses a time step of $dt = 0.1$ time units and simulates for $t_{\text{max}} = 30$ time units. Let the frictional forces be negligible.

At the end, the program plots the position x , the total energy E and the energy components over the time.

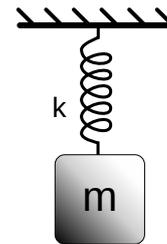


Figure 1: Spring Pendulum

- 1.1.1 (2 points) Encapsulate the simulation of the pendulum into a class `Pendulum` that looks as follows:

```
class Pendulum:
    def __init__(self, k, dt, x_init = 0.0, v_init = 0.0):
        # fill in
    def step(self):
        """Perform a single simulation time step."""
        # fill in
    def getEnergy(self):
        """Compute and return the energy components."""
        # fill in
    def simulate(self, tmax):
        """Simulate the pendulum up to tmax.
        Return the measurements as a NumPy array."""
        # fill in
```

- 1.1.2 (2 points) Extend the program such that it simulates two pendulums, one with $k = 1.0$ and $x_0 = 0.3$, the other with $k = 0.7$ and $x_0 = 0.1$. Let the program create a figure that shows four subplots:
 - the positions x of both pendulums vs. time
 - the total energy E of both pendulums vs. time
 - the kinetic energy E_{kin} of both pendulums vs. time
 - the potential energy E_{pot} of both pendulums vs. time

Hint

Use `pydoc matplotlib.pyplot.subplot` to get help on how to create the sub plots

Task 1.2 (6 points): Coupled Spring Pendulum

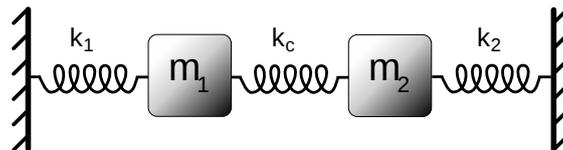


Figure 2: Coupled Spring Pendulum (Sketch¹ by jim.belk, CC-BY-SA 3.0)

In this task, the program from the previous task should be extended to simulate a coupled spring pendulum. A coupled spring pendulum consists of two masses $m_1 = m_2 = 1$, that are fixed between walls with three springs (spring constants $k_1 = 1$, $k_2 = 0.7$, $k_c = 0.2$) as in figure 2. Let the gravitation and frictional forces be negligible.

¹http://en.wikipedia.org/wiki/File:Coupled_Harmonic_Oscillator.svg

Initially, the masses are at rest ($v_1(0) = v_2(0) = 0$) and both are displaced from their equilibrium positions ($x_1(0) = 0.3$ and $x_2(0) = 0.2$). The forces acting on both masses are $F_1 = -k_1x_1 - k_c(x_1 - x_2)$ and $F_2 = -k_2x_2 - k_c(x_2 - x_1)$. The “energy of mass i ” is $E_i = \frac{1}{2}k_ix_i^2 + \frac{1}{2}m_iv_i^2$, the energy that is stored in the coupling spring is $E_c = \frac{1}{2}k_c(x_1 - x_2)^2$. The total energy of the system is $E = E_1 + E_2 + E_c$.

- 1.2.1 (4 points) Extend the program to simulate the coupled spring pendulum. Let the program create a figure with three subplots that shows
 - the positions x of both masses vs. time
 - the energies of both masses E_i vs. time
 - the total energy E of the system vs. time
- 1.2.2 (2 points) Modify the algorithms as follows: Change the method `step` such that it first computes the new positions from the old velocities, and only then the new forces and velocities. Let the program create a plot of the total energy of the system both for the original algorithm (the *Symplectic Euler algorithm*) as well as for the modified algorithm (the *Euler forward algorithm*).

What is the difference between the algorithms? Which of both algorithms would you prefer for an actual simulation?

Hint The length of the springs is not required as long as the equilibrium length is larger than the maximal displacement.