Worksheet 1: Python and NumPy

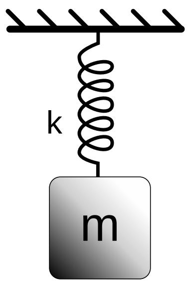
9th April 2014

General Remarks

- The deadline for the worksheets is **Wednesday, 16 April 2014, 10:00** for the tutorial on Friday and **Friday, 18 April 2014, 10:00** for the tutorials on Tuesday and Wednesday. .
- On this worksheet, you can achieve a maximum of 10 points.
- To hand in your solutions, send an email to
  - Johannes (`zeman@icp.uni-stuttgart.de`; Tuesday, 9:45–11:15)
  - Tobias (`richter@icp.uni-stuttgart.de`; Wednesday, 15:45–17:15)
  - Shervin (`shervin@icp.uni-stuttgart.de`; Friday, 11:30–13:00)
- Attach all required files to the mailing. If asked to write a program, attach the *source code* of the program. If asked for a text, send it as PDF or in the text format. We will *not* accept MS Word files!
- The worksheets are to be solved in groups of two or three people. We will not accept hand-in-exercises that only have a single name on it.
- The tutorials take place in the CIP-Pool of the Institute for Computational Physics (ICP) in Allmandring 3.

Task 1.1: Spring Pendulum (6 points)

Copy the IPython Notebook `pendulum.ipynb` or the Python program `pendulum.py` from the home page to your home directory.



The program simulates a spring pendulum, *i.e.* a mass ($m = 1$) that is coupled to a harmonic spring (spring constant $k = 1$), as shown in the figure. Initially, the mass is displaced from the equilibrium position ($x_0 = 0.3$) and at rest ($v_0 = 0$). The force that acts on the mass is $F = -kx$, the potential energy is $E_{\text{pot}} = \frac{1}{2}kx^2$ and the kinetic energy $E_{\text{kin}} = \frac{1}{2}mv^2$. To simulate the pendulum, it uses a time step of $dt = 0.1$ and simulates for $t_{\text{max}} = 10$. Let the frictional forces be negligible. At the end, the program plots the position x, the total energy E and the energy components over the time.

- **1.1.1** (2 points) Encapsulate the simulation of the pendulum into a class `Pendulum` that looks as follows:

```python
class Pendulum:
  def __init__(self, k, dt, x0 = 0.0, v0 = 0.0):
      # fill in
  def step(self):
      """Perform a single simulation time step."""
      # fill in
  def getEnergy(self):
      """Compute and return the energy components."""
      # fill in
  def simulate(self, tmax):
      """Simulate the pendulum up to tmax.
      Return the measurements as a NumPy array."""
      # fill in
```

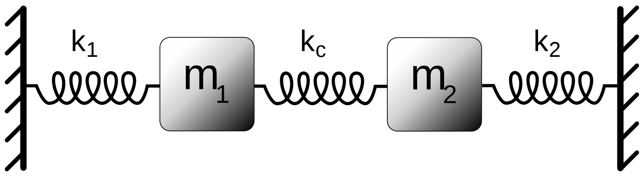
- **1.1.2** (2 points) Extend the program such that it simulates two pendulums, one with $k = 1.0$ and $x_0 = 0.3$, the other with $k = 0.7$ and $x_0 = 0.1$. Let the program create a figure that shows four subplots:
  - the positions x of both pendulums vs. time
  - the total energy E of both pendulums vs. time
  - the kinetic energy E_{kin} of both pendulums vs. time
  - the potential energy E_{pot} of both pendulums vs. time

  Hint Use `pydoc matplotlib.pyplot.subplot` to get help on how to create the sub plots, or have a look at the matplotlib gallery at `http://matplotlib.org/gallery.html`.

- **1.1.3** (2 points) Change the method `step` such that it first computes the new positions from the old velocities, and only then the new velocities (the forces are still to be computed at the beginning of the method). Let the program create a plot of the total energy of the system both for the original algorithm (the *Symplectic Euler algorithm*) as well as for the modified algorithm (the *Euler forward algorithm*) and perform the simulation for $t_{\max} = 30$.

  What is the difference between the algorithms? Which of both algorithms would you prefer for an actual simulation?

Task 1.2: Coupled Spring Pendulum (4 points)



Now, the program from the previous task should be extended to simulate a *coupled* spring pendulum. A coupled spring pendulum consists of two masses $m_1 = m_2 = 1$, that are fixed between walls with three springs (spring constants $k_1 = 1$, $k_2 = 0.7$, $k_c = 0.2$) as in the figure above[1]. Let the gravitation and frictional forces be negligible.

Initially, the masses are at rest ($v_1(0) = v_2(0) = 0$) and both are deplaced from their equilibrium positions ($x_1(0) = 0.3$ and $x_2(0) = 0.2$). The system is simulated for $t_{\max} = 30$ time units. The forces acting on both masses are $F_1 = -k_1 x_1 - k_c(x_1 - x_2)$ and $F_2 = -k_2 x_2 - k_c(x_2 - x_1)$. The "energy of mass i" is $E_i = \frac{1}{2}k_i x_i^2 + \frac{1}{2}m_i v_i^2$, the energy that is stored in the coupling spring is $E_c = \frac{1}{2}k_c(x_1 - x_2)^2$. The total energy of the system is $E = E_1 + E_2 + E_c$.

Extend the program to simulate the coupled spring pendulum for 30 time units. Let the program create a figure with three subplots that shows

- the positions x of both masses vs. time
- the energies of both masses E_i vs. time
- the total energy E of the system vs. time

Hint The lengths of the springs is not required as long as the equilibrium length is larger than the maximal displacement.

[1]Sketch by `jim.belk`, CC-BY-SA 3.0, `http://en.wikipedia.org/wiki/File:Coupled_Harmonic_Oscillator.svg`