Problem Set #5

Problem 1

1) $$\mu = \frac{Q}{6\pi\eta R} = \frac{1.602\times10^{-19}\,C}{6\pi(8.94\times10^{-4}\,kg/ms)(1\,\text{Å})} = 9.5\times10^{-8}\,\frac{cs}{kg}$$

2) $$\mu = \frac{\sigma}{\eta K} = \frac{0.2\,e/nm^2\,(1.602\times10^{-19}\,C/e)}{\left(\frac{0.304\,nm}{\sqrt{0.1}}\right)^{-1}\left(8.94\times10^{-4}\,\frac{kg}{ms}\right)} = 3.5\times10^{-8}\,\frac{cs}{kg}$$

3) $$D = \frac{kT}{6\pi\eta R} = \frac{4.11\times10^{-21}\,J}{6\pi(8.94\times10^{-4}\,\frac{kg}{ms})(1\,\text{Å})} = 2.4\times10^{-9}\,m^2/s \quad (N_a)$$

$$D = \frac{4.11\times10^{-21}\,J}{6\pi(8.94\times10^{-4}\,\frac{kg}{ms})(1\times10^{-6}\,m)} = 2.4\times10^{-13}\,m^2/s \quad (Coll_{o.d})$$

4) $$\mu = \frac{x}{tE} \Rightarrow t = \frac{x}{\mu E} \qquad D = \frac{x^2}{2t} \Rightarrow t = \frac{x^2}{2D}$$

$$\frac{x}{\mu E} = \frac{x^2}{2D} \Rightarrow x = \frac{2D}{\mu E},\quad t = \frac{2D}{(\mu E)^2}$$

N_o) $$x = \frac{2D}{\mu E} = \frac{2(2.4\times10^{-9}\,m^2/s)}{(9.5\times10^{-8}\,\frac{cs}{kg})(10\,V/cm)} = 5\times10^{-5}\,m$$

$$t = \frac{2D}{(\mu E)^2} = \frac{2(2.4\times10^{-9}\,m^2/s)}{[9.5\times10^{-8}\,\frac{cs}{kg}\;10\,V/cm]^2} = 0.532\,s$$

Coll$_{o.d}$) $$x = \frac{2D}{\mu E} = \frac{2(2.4\times10^{-13}\,m^2/s)}{(3.5\times10^{-8}\,\frac{cs}{kg})(10\,V/cm)} = 1.37\times10^{-8}\,m$$

$$t = \frac{2D}{(\mu E)^2} = \frac{2(2.4\times10^{-13}\,m^2/s)}{[3.5\times10^{-8}\,\frac{cs}{kg}\;10\,V/cm]^2} = 0.00039\,s$$

Problem 2

1) $F = QE = 6\pi\eta R v$

$$v = \frac{QE}{6\pi\eta R}$$

2) $\zeta = \frac{Q}{4\pi\varepsilon R} e^{-kR} \approx \frac{Q}{4\pi\varepsilon R}$

$$\mu = \frac{v}{E} = \frac{Q}{6\pi\eta R} = \frac{\zeta 4\pi\varepsilon R}{6\pi\eta R}$$

$$\mu = \frac{2\zeta\epsilon}{3\eta}$$

Problem 3

1) $\nabla^2\phi = -\frac{\rho_e}{\epsilon} = -\sum_i \frac{z_i e n_{bi} \exp\left(\frac{-z_i e\phi}{kT}\right)}{\epsilon}$

2) $\nabla^2\phi = -\sum_i \frac{z_i e n_{oi}}{\epsilon} + \sum_i \frac{z_i^2 e^2 n_{oi}}{\epsilon kT}\phi$

$$\nabla^2\phi = \sum_i \frac{z_i^2 e^2 n_{oi}}{\epsilon kT}\phi$$

$$\nabla^2\phi = k^2\phi \quad , k = \left[\sum_i \frac{z_i^2 e^2 n_{oi}}{\epsilon kT}\right]^{1/2}$$

3) $\phi = Ae^{ky} + Be^{-ky}$

$$\phi = \zeta e^{-ky}$$

$$E = -\nabla\phi = -\zeta k = \frac{-\sigma}{\epsilon} \Rightarrow \zeta = \frac{-\sigma}{\epsilon k}$$

$$\phi = \frac{-\sigma}{\epsilon k} e^{-ky}$$

4) $\eta \nabla^2 v = -E\rho_e$

$\eta \nabla^2 v = \epsilon E \nabla^2 \phi$

$\eta \nabla^2 v = \epsilon E \frac{\sigma}{\epsilon k} k^2 e^{-ky}$

$\nabla^2 v = -\frac{E\sigma k e^{-ky}}{\eta}$

$v = C_1 + C_2 e^{-ky}$, $v(y=0)=0 \Rightarrow C_2 = -C_1$

$v = [1 - e^{-ky}] C_3$

$\eta \nabla^2 v = -C_3 k^2 e^{-ky} = -\frac{E\sigma k}{\eta} e^{-ky}$

$C_3 = +\frac{E\sigma}{\eta k}$

$v = [1 - e^{-ky}] \frac{E\sigma}{\eta k}$

$v(k_y \gg 1) = \frac{E\sigma}{\eta k} = \frac{E(\zeta \epsilon k)}{\eta k}$

$\mu = \frac{v(k_y \gg 1)}{E} = \frac{\zeta \epsilon}{\eta}$